13/05/15

Titolo nota 13/05/2015

1) $3x^2 - 4\sqrt{3}x + 4 \leq 0$

$a = 3$

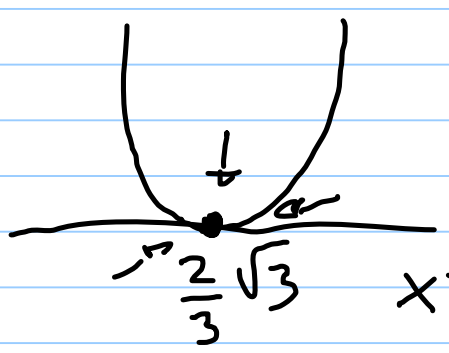
$b = -4\sqrt{3}$

$c = 4$

$\Delta = b^2 - 4ac = (-4\sqrt{3})^2 - 4 \cdot 3 \cdot 4$

$= 16 \cdot 3 - 4 \cdot 3 \cdot 4 = 0$ due sol. coincidenti

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{4\sqrt{3} \pm \sqrt{0}}{6} = \frac{4\sqrt{3}}{6} = \frac{2}{3}\sqrt{3}$



2) $x^5 - 7x^3 + 12x > 0$

$x\,(x^4 - 7x^2 + 12) > 0$

1F 2F

$\frac{N}{D} \leq 0$

$1F > 0 \qquad x > 0$

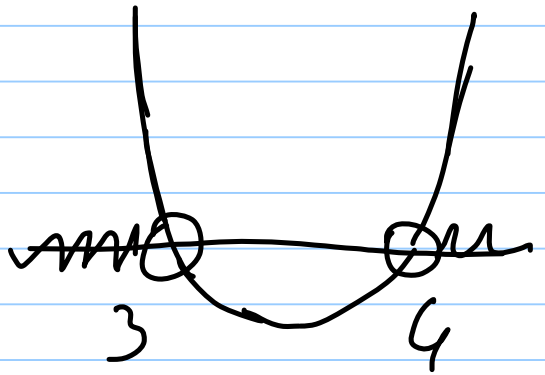
$2F > 0$ b) $x^4 - 7x^2 + 12 > 0$

b) $x^4 - 7x^2 + 12 > 0$ $\qquad t = x^2$

$t^2 - 7t + 12 > 0$

$\Delta = b^2 - 4ac = 49 - 4 \cdot 12 = 1$

$t_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{7 \pm 1}{2}$ < 4, 3

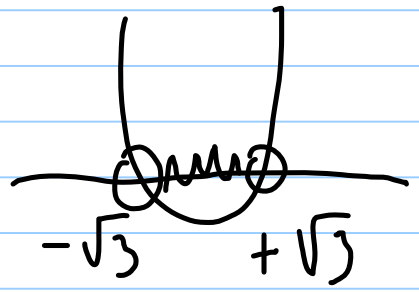


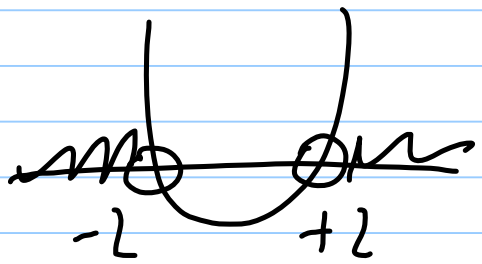
$t < 3 \quad \vee \quad t > 4$

$x^2-3<0$ $\quad x^2<3$ $\quad \vee \quad x^2>4$ $\quad x^2-4>0$

$x^2=3$ $\qquad x^2=4$

$x=\pm\sqrt{3}$ $\qquad x=\pm 2$



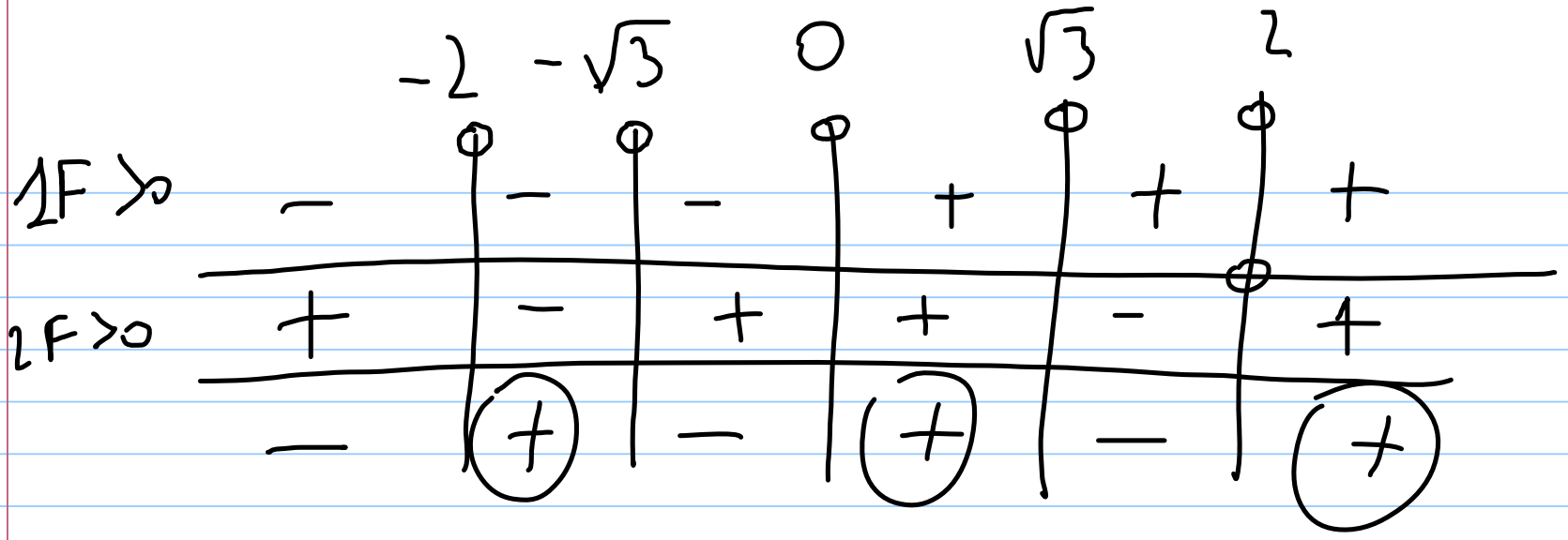


$-\sqrt{3}<x<\sqrt{3}$ $\quad \vee \quad x<-2 \quad \vee \quad x>2$

$x<-2 \quad \vee \quad -\sqrt{3}<x<\sqrt{3} \quad \vee \quad x>2$

1F >0 $\quad x>0$

2F >0 $\quad x<-2 \vee -\sqrt{3}<x<\sqrt{3} \quad x>2$



$-2 < x < -\sqrt{3} \quad \vee \quad 0 < x < \sqrt{3} \quad \vee \quad x > 2$

) $x^3 - 2x^2 - x + 2 \geq 0$

$x^2(x-2) - 1(x-2) \geq 0$

$(x-2)(x^2-1) \geq 0$

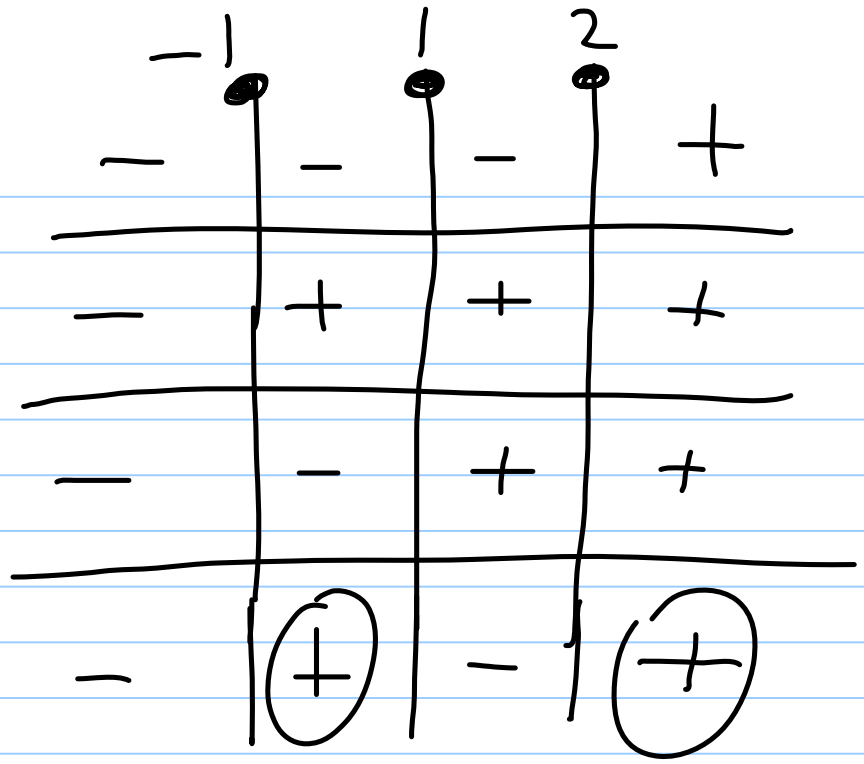
$(x-2)(x+1)(x-1) \geq 0$

1F 2F 3F

$1F \geq 0 \qquad x-2 \geq 0 \qquad x \geq 2$

$2F \geq 0 \qquad x+1 \geq 0 \qquad x \geq -1$

$3F \geq 0 \qquad x-1 \geq 0 \qquad x \geq 1$



$-1 \leq x \leq 1 \quad \vee \quad x \geq 2$

) $\dfrac{x+1}{x-1} \geq \dfrac{1}{2}$ $\quad c\in \quad x-1 \neq 0$

$x \neq 1$

$\dfrac{x+1}{x-1} - \dfrac{1}{2} \geq 0$

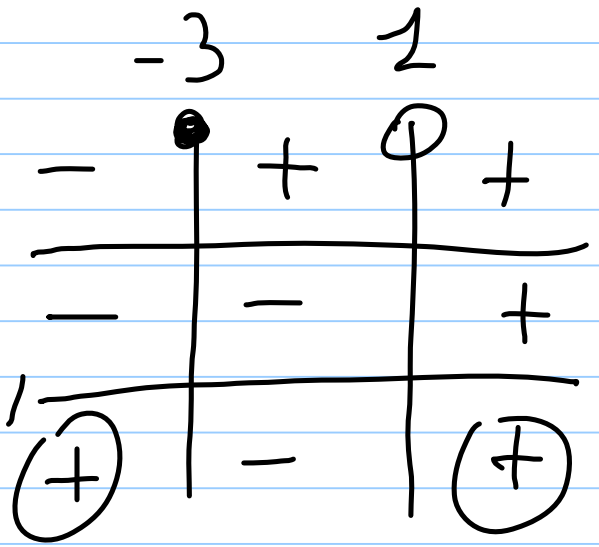
$$\frac{2(x+1)-(x-1)}{2(x-1)} \geq 0$$

$$\frac{2x+2-x+1}{2(x-1)} \geq 0$$

$$\frac{x+3}{2(x-1)} \geq 0$$

$N \geq 0 \quad x+3 \geq 0 \quad x \geq -3$

$D > 0 \quad 2(x-1) > 0 \quad x-1 > 0 \quad x > 1$



$x \leq -3 \ \vee \ x > 1$

$$\sqrt[3]{x^3+2} < x-1$$

$$x^3+2 < x^3-3x^2+3x-1$$

$$\frac{3x^2-3x+3}{3} < \frac{0}{3}$$

$$\boxed{x^2-x+1<0}$$

$\Delta = 1-4 < 0$ mess. soluzione

$\sqrt[3]{3}$ $\sqrt[3]{-3}$

$\sqrt[2]{4}$ $\sqrt[2]{-4}$

$(A+B)^3 = A^3+3A^2B+3AB^2+B^3$

$a>0$

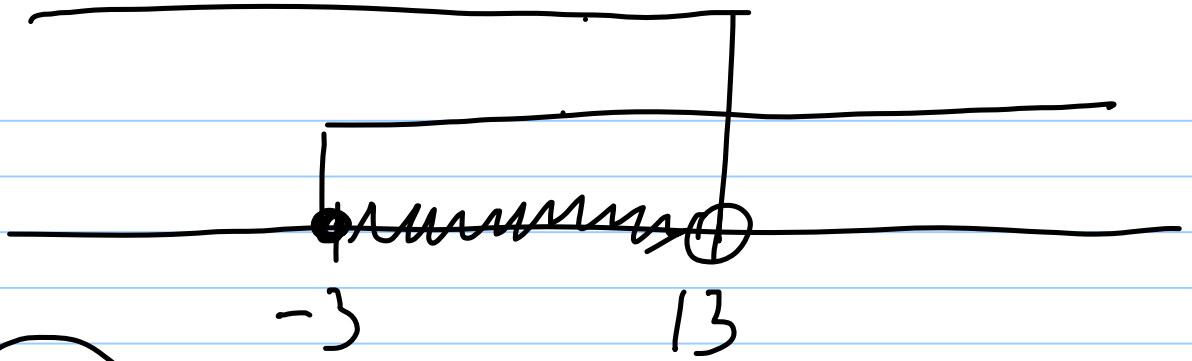
$\cup$

$\nexists x \in \mathbb{R}$

6) $\sqrt{x+3} < 4$

$$\begin{cases} x+3 \geq 0 \\ \left(\sqrt{x+3}\right)^2 < (4)^2 \end{cases} \quad \begin{cases} x \geq -3 \\ x+3 < 16 \end{cases} \quad \begin{cases} x \geq -3 \\ x < 13 \end{cases}$$

$-3 \leq x < 13$



$\sqrt{x+3} < -4$ ←

$\emptyset$

$x+3 \geq 0$

$x \geq -3$

$\sqrt{x+3} > -4$ $\quad x \geq -3$

$\sqrt{1-x} \leq 1+x$ ←

$+ \qquad -$

$$\begin{cases} 1-x \geq 0 \; + \\ 1+x \geq 0 \; + \\ (\sqrt{1-x})^2 \leq (1+x)^2 \; + \end{cases}$$

$$0 \leq x \leq 1$$

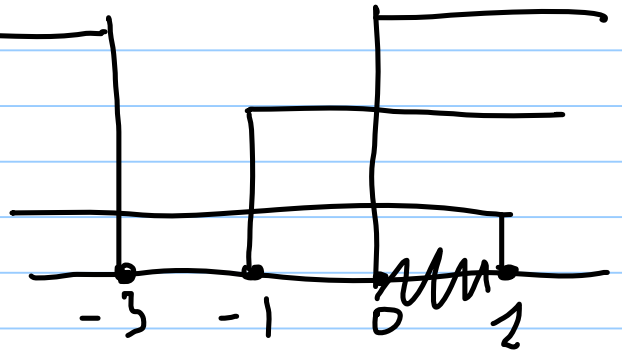
$$\vee$$

$$\begin{cases} 1-x \geq 0 \; + \\ 1+x \leq 0 \; + - \\ \text{mess sol} \end{cases}$$

$$\begin{cases} x \leq 1 \\ x \geq -1 \\ 1-x \leq 1+2x+x^2 \end{cases}$$

c)

$$\begin{cases} x \leq 1 \\ x \geq -1 \\ x \leq -3 \vee x \geq 0 \end{cases}$$



c) $1 - x \leq 1 + 2x + x^2$

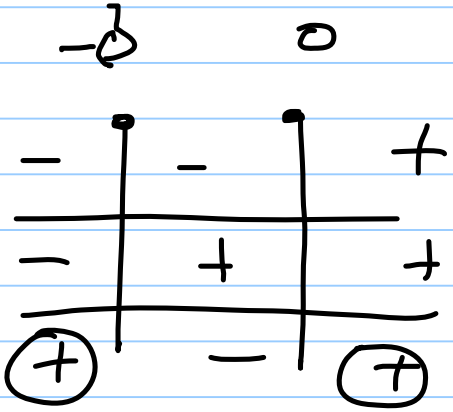
$-x^2 - 3x \leq 0$

$x^2 + 3x \geq 0$

$x(x+3) \geq 0$

$1F \geq 0 \quad x \geq 0$

$2F \geq 0 \quad x \geq -3$

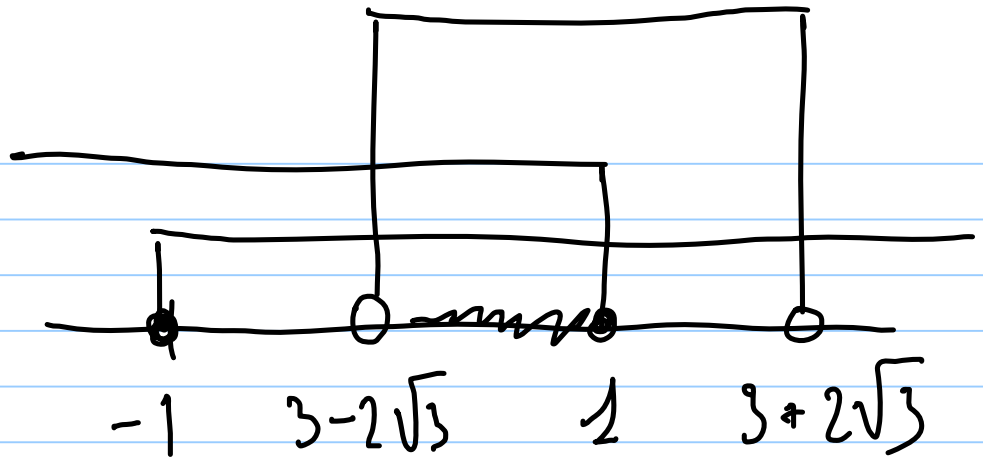


) $2\sqrt{x+1} > 1-x$

$$\begin{cases} x+1 \geq 0 \\ 1-x \geq 0 \\ (2\sqrt{x+1})^2 > (1-x)^2 \end{cases} \quad \cup \quad \begin{cases} x+1 \geq 0 \\ 1-x < 0 \\ \forall x \end{cases}$$

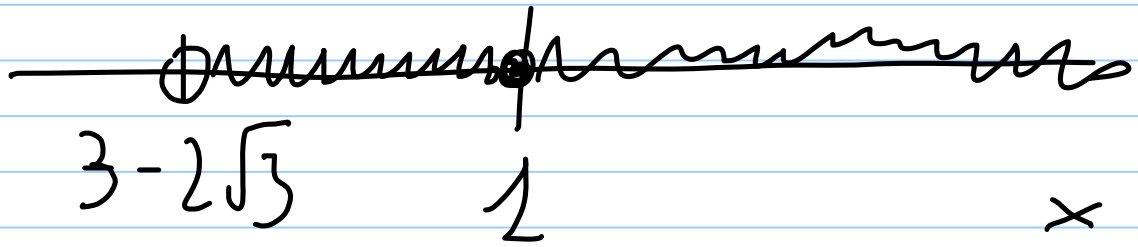
c) $$\begin{cases} x \geq -1 \\ -x \geq -1 \\ 4x+4 > 1-2x+x^2 \end{cases} \quad \cup \quad \begin{cases} x+1 \geq 0 \\ 1-x < 0 \end{cases}$$

$$\begin{cases} x \geq -1 \\ x \leq 1 \\ 3-2\sqrt{3} < x < 3+2\sqrt{3} \end{cases} \quad \cup \quad \begin{cases} x \geq -1 \\ x > 1 \end{cases}$$



$x > 1$

$3-2\sqrt{3} < x \leq 1$ $\quad \cup \quad$ $x > 1$



$x > 3-2\sqrt{3}$

Grafici deducibili

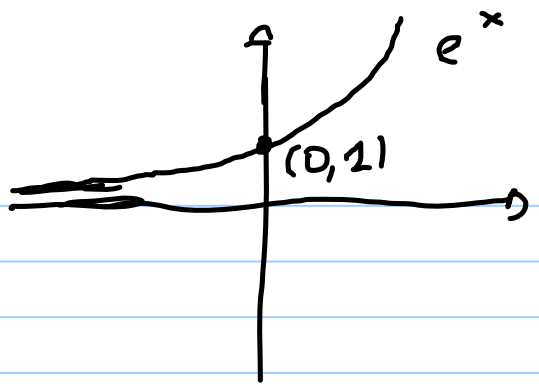
$f(x)$

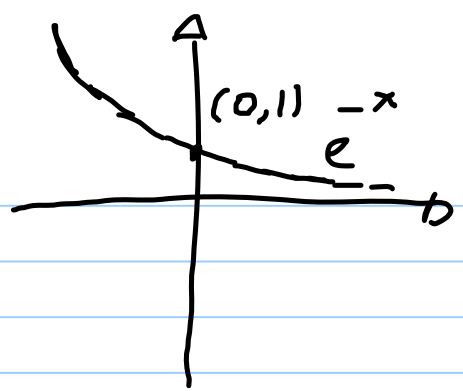
- $f(x+k)$ traslo a dx o a sx
- $k + f(x)$ traslo in alto o in basso
- $f(-x)$ simmetrico rispetto asse y
- $-f(x)$ simmetrico rispetto asse x
- $|f(x)|$

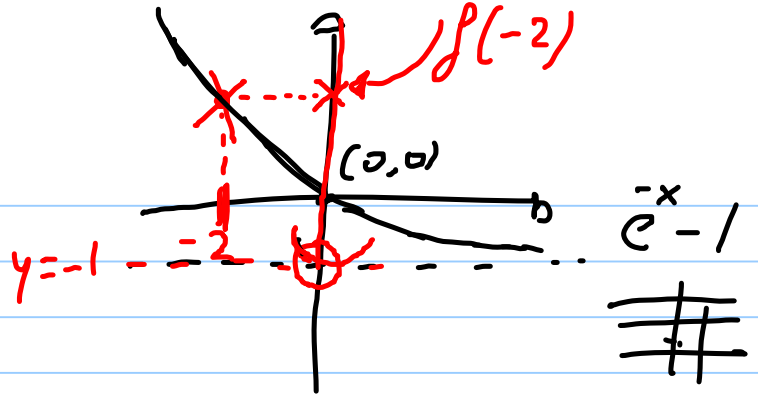
$f(x) = e^{-x} - 1$

$e^{x} \rightarrow e^{-x} \rightarrow e^{-x} - 1$

$f(x) \rightarrow f(-x) \rightarrow f(-x) - 1$



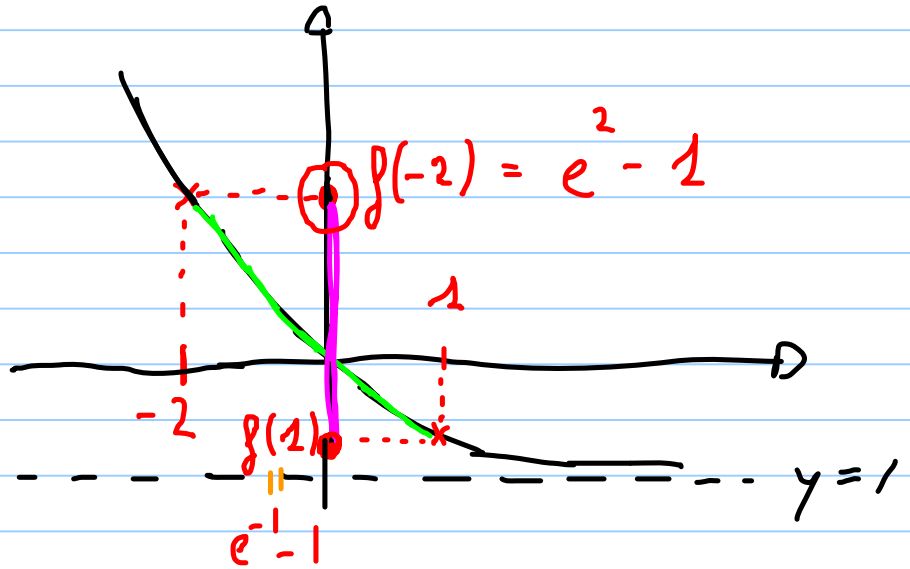




D: $\forall x$

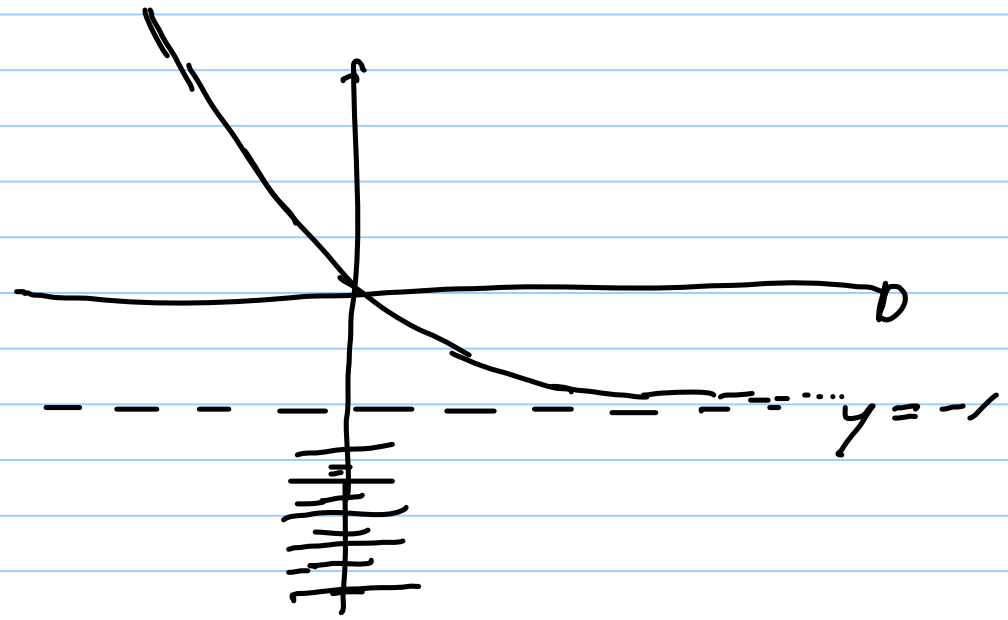
$\mathcal{C} = \text{Im}\, f = (-1, +\infty) =$

Immagine di -2 $\quad f(-2) = e^2 - 1$

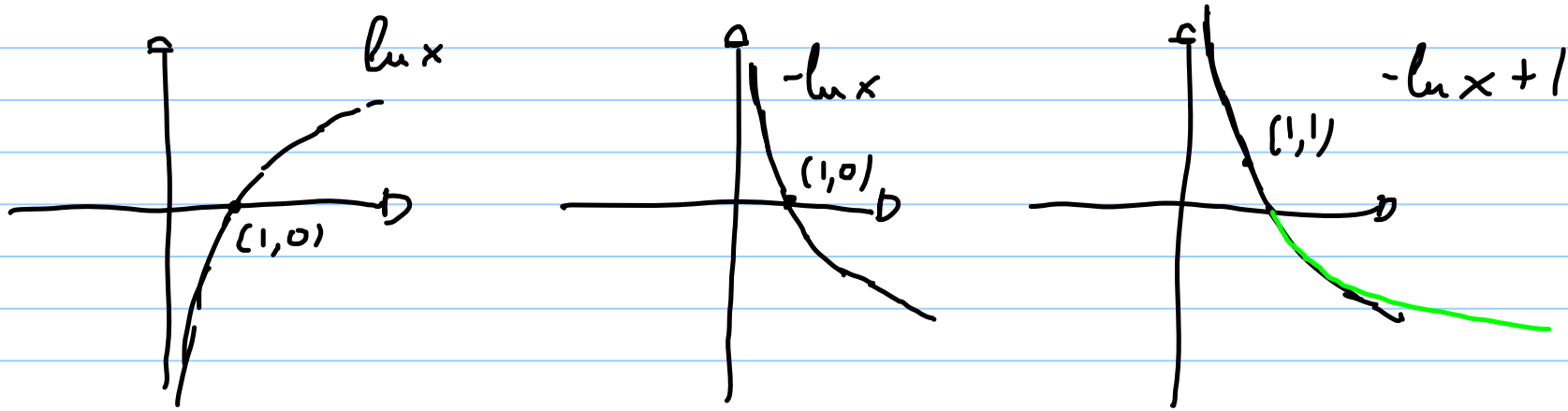


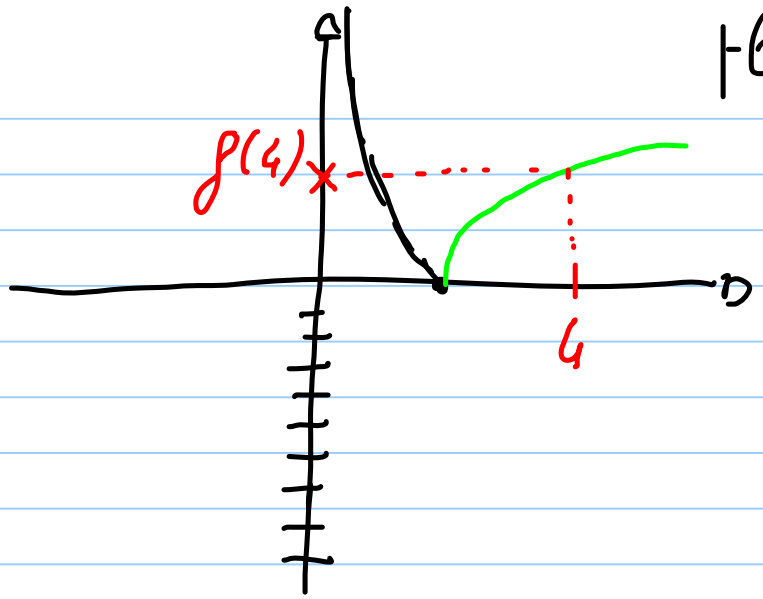
$f(1) = e^{-1} - 1$

$$\text{Imm}(-2, 1] = \left[e^{-1} - 1, e^{2} - 1\right)$$



) $f(x) = |1 - \ln(x)|$ $\ln(x) \to -\ln(x) \to -\ln x + 1 \to |-\ln x + 1|$





$|\ln x+1|$

$\Omega : x > 0$

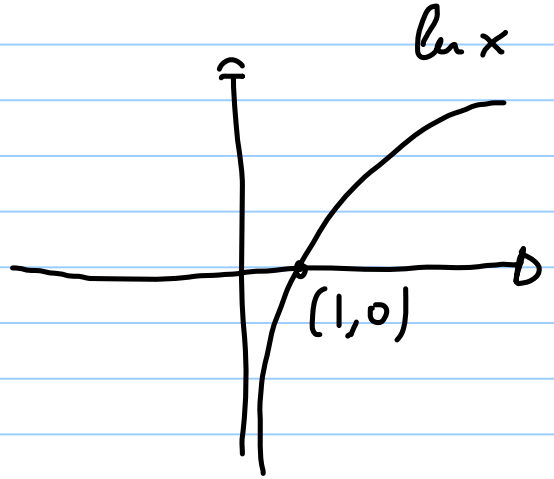
$\boxed{\mathcal{C} : [0, +\infty)}$

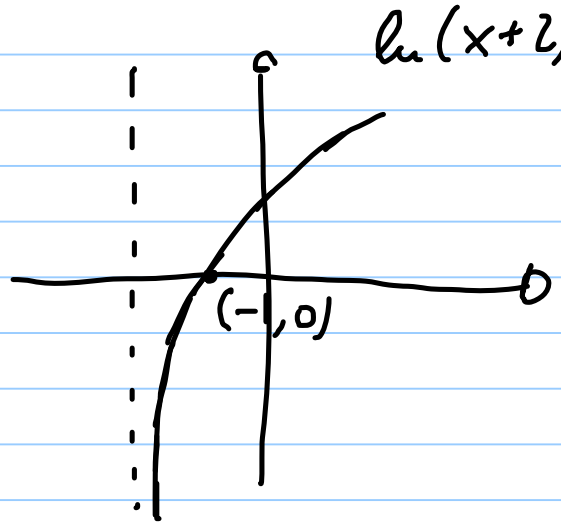
$f(4) = |-\ln 4 + 1|$

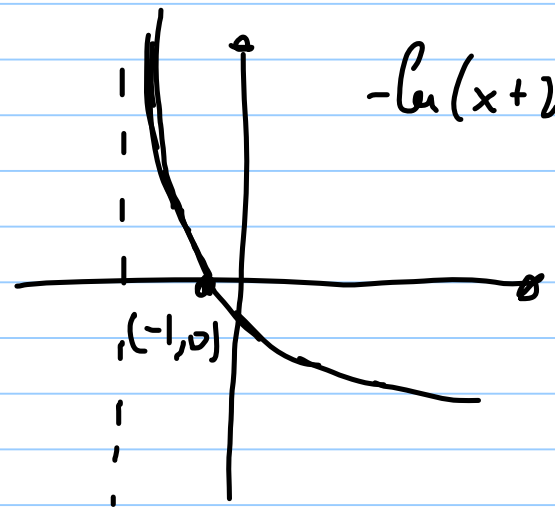
) $f(x) = -\ln(x+2)$

$\ln x \to \ln(x+2) \to -\ln(x+2)$

$f(x) \to f(x+2) \to -f(x+2)$



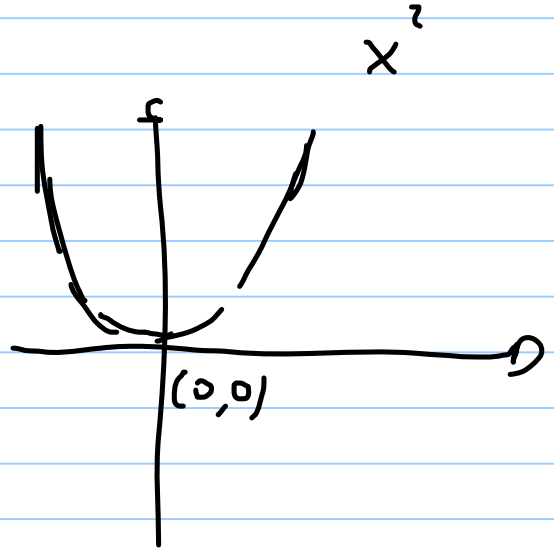


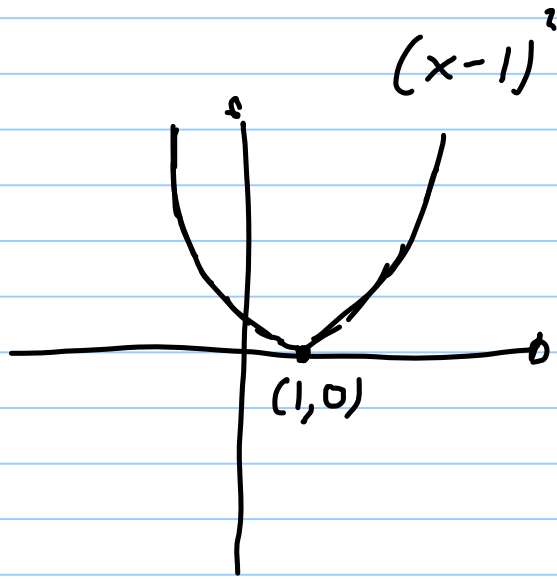


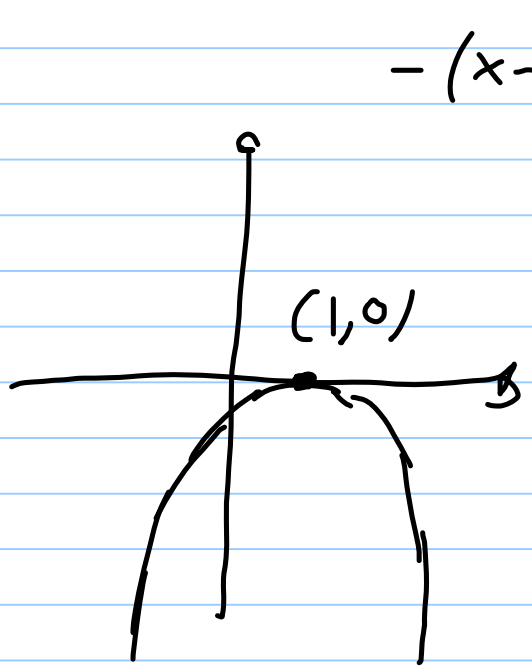
) $f(x) = 1 - (x-1)^2$

$x^2 \to (x-1)^2 \to -(x-1)^2$

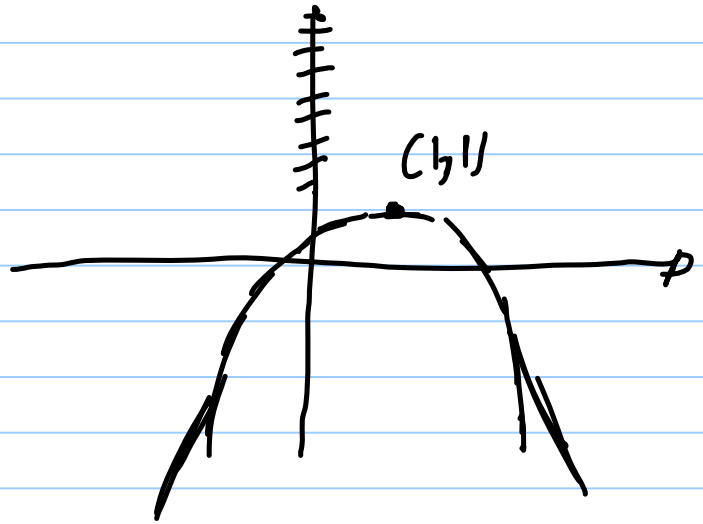
$\to -(x-1)^2 + 1$







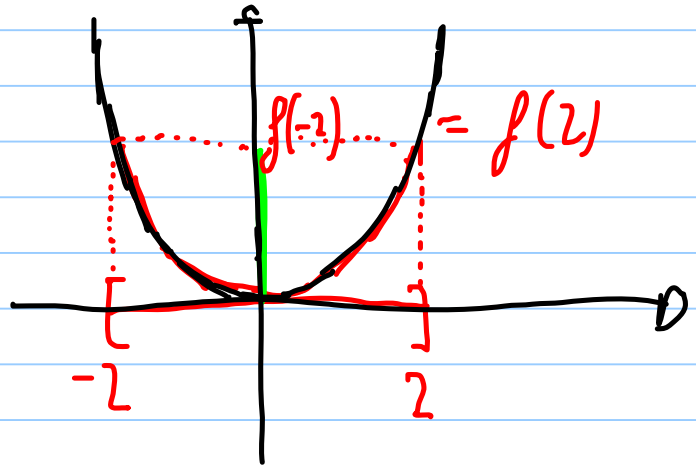
$-(x-1)^2 + 1$



$D : \mathbb{R}$

$C : (-\infty, 1]$

$f(x) = x^2$? = $\mathrm{Im}\,[-2, 2] = [0, 4]$



$0 \le y \le f(-2)$

$0 \le y \le f(2)$

$0 \le y \le 4$