10/06/15

Titolo nota 10/06/2015

Es:

$$v^1 = (0,-1,1) \qquad v^2 = (1,2,-1) \qquad v^3 = (-1,1,0)$$

$$v^1, v^2, \dots, v^n \qquad \alpha_1 v^1 + \alpha_2 v^2 + \dots + \alpha_n v^n = 0 \qquad \alpha_1 = \alpha_2 = \dots = \alpha_n = 0$$

$$a v^1 + b v^2 + c v^3 = 0$$

$$a \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + c \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} a \cdot 0 + b \cdot 1 + c(-1) = 0 \\ a(-1) + b \cdot 2 + c \cdot (1) = 0 \\ a \cdot 1 + b(-1) + c \cdot 0 = 0 \end{cases} \qquad \begin{cases} b - c = 0 \\ -a + 2b + c = 0 \\ a - b \neq 0 \end{cases}$$

$$\begin{cases} b = c \\ -\cancel{c}+2c+\cancel{c} = 0 \\ a = b = c \end{cases} \qquad \begin{cases} b = c \\ 2c = 0 \\ a = b = c \end{cases} \qquad \begin{cases} b = 0 \\ c = 0 \\ a = 0 \end{cases} \qquad a = b = c = 0 \quad \text{l. ind.}$$

$$A = \begin{array}{c} \begin{array}{ccc} v^1 & v^2 & v^3 \end{array} \\ \begin{pmatrix} 0 & 1 & -1 \\ -1 & 2 & 1 \\ \boxed{1} & -1 & 0 \end{pmatrix} \end{array}$$

$$\det A = +1 \cdot \det \begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} + 1 \det \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} =$$

$$= 1\,[0-1] + 1\,[1-(-2)] =$$

$$= 1\cdot(-1) + [1+2] = -1+3 = 2 \neq 0$$

$$\mathcal{B} = \left\{ \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \cancel{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}} \right\}$$

$$e^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \text{SI}$$

$$a v^1 + b v^2 + \cancel{c v^3} = e^1$$

$$a \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + c \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} b - c = 1 \\ -a + 2b + c = 0 \\ a - b = 0 \end{cases}$$

Es:

$$\overset{3\times 4}{A} = \begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 1 \end{pmatrix}$$

$$A' = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix} \qquad \det A' = 1 \cdot \det\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} = 1\,[0-1] = -1 \neq 0$$

$\det A' \neq 0 \Rightarrow r(A) = 3 \Rightarrow$ le righe sono l. ind.

le righe formano sottospazio di $\mathbb{R}^4$ di dimensione 3. Una base di questo sottospazio è

$$\mathcal{B} = \{(0,1,-1,0);\ (1,-1,0,0);\ (1,0,-1,1)\}$$

Le colonne sono l. d.. Esse formano sottospazio di $\mathbb{R}^3$ di dimensione 3. Una base di questo sottospazio è

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \qquad e^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$Ax = 0$$

$$\underset{A}{\begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 1 \end{pmatrix}} \underset{\bar{x}}{\begin{pmatrix} x \\ y \\ \underset{z}{w} \end{pmatrix}} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} y - w = 0 \\ x - y = 0 \\ x - w + z = 0 \end{cases} \qquad \begin{cases} y = w \\ x = y = w \\ w - w + z = 0 \end{cases} \qquad \begin{cases} y = w \\ x = w \\ z = 0 \end{cases}$$

$$\text{Sol:} \quad \begin{array}{cccc} x & y & w & z \\ \end{array}$$
$$\left(w, w, w, 0 \right)$$

Es:

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & -1 & 1 \end{pmatrix}$$

$$A' = \begin{pmatrix} 1 & 1 & \cancel{0} \\ 1 & 0 & \cancel{0} \\ -2 & -1 & \cancel{1} \end{pmatrix} \qquad \det A' = 1 \cdot \det \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = 1 \cdot (-1) = -1 \neq 0$$

$\det A' \neq 0 \Rightarrow r(A) = 3 \Rightarrow$ le righe sono l. ind. Esse formano sottospazio S di $\mathbb{R}^4$ di dimensione 3. Una base di questo sottospazio è

$$\mathcal{B} = \Big\{ \underbrace{(0, 1, 1, 0)}_{x^1};\ \underbrace{(-1, 1, 0, 0)}_{x^2};\ \underbrace{(1, -2, -1, 1)}_{x^3} \Big\}$$

$(1, 0, 0, 0)$

$$a x^1 + b x^2 + c x^3 = (1, 0, 0, 0)$$

$$a \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + c \begin{pmatrix} 1 \\ -2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -b + c = 1 \\ a + b - 2c = 0 \\ a - c = 0 \\ c = 0 \end{cases} \qquad \begin{cases} 0 = 1 \\ b = -a + 2c = 0 \\ a = c = 0 \\ c = 0 \end{cases} \quad \text{impossibile}$$

$(1, 0, 0, 0) \notin S$ generato da x^1, x^2, x^3

Le colonne sono l. d.

Le colonne l. ind. sono le 2°, 3°, 4°

Le colonne formano sottospazio di $\mathbb{R}^3$ di dimensione 3.

Una base è

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$(2, 0, -1) \in S_{\text{colonne}}$

ES:

$\mathcal{C}: (x-x_0)^2 + (y-y_0)^2 = r^2$

$C(x_0, y_0) \quad r$

$$x^2 + y^2 - 4x = 0$$

$$x^2 - 4x + y^2 = 0$$

$$\underbrace{x^2 - 4x + 4} - 4 + y^2 = 0$$

$$(x-2)^2 - 4 + y^2 = 0$$

$$(x-2)^2 + y^2 = 4$$

$\mathcal{C}$ $\quad C(2,0) \quad r = 2$

$$x^2 + y^2 - 4x - 8y = 0$$

$$x^2 - 4x + y^2 - 8y = 0$$

$$\underbrace{x^2 - 4x + 4} - 4 + \underbrace{y^2 - 8y + 16} - 16 = 0$$

$$(x-2)^2 + (y-4)^2 = 20$$

$\mathcal{C}$ $\quad C(2,4) \quad r = \sqrt{20} = 2\sqrt{5}$

$$\mathcal{P}: \begin{cases} y = (x - x_0)^2 + y_0 \\ x = (y - y_0) + x_0 \end{cases}$$

$V(x_0, y_0)$

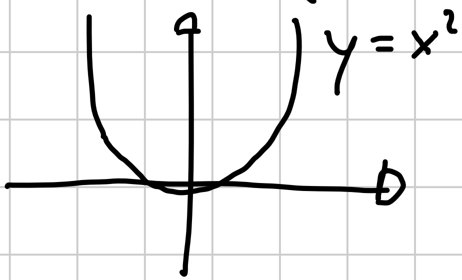
$$\boxed{y > x^2 + 4x + 7}$$

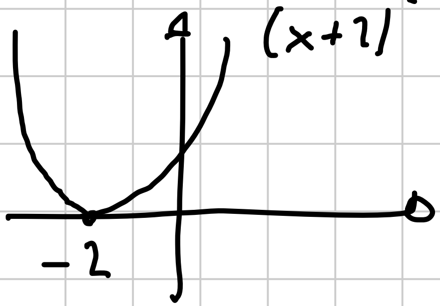
$$y = x^2 + 4x + 7$$

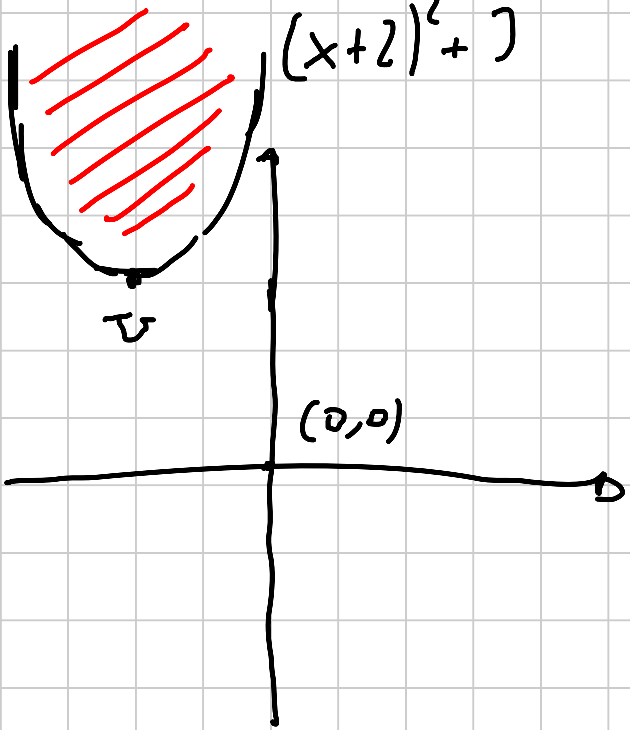
$$y = x^2 + 4x + 4 - 4 + 7$$

$$\rightarrow y = (x+2)^2 + 3 \leftarrow$$

$V(-2, 3)$







$$y > x^2 + 4x + 7 \qquad (0,0)$$

$0 > 0^2 + 4 \cdot 0 + 7$

$0 > 7$ falsa

8:

$x > -y^2 + 8y - 14$

$x = -y^2 + 8y - 14$

$x = -(y^2 - 8y) - 14$

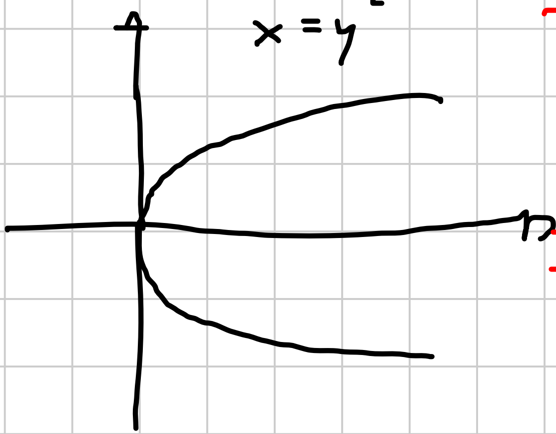
$x = -1(y^2 - 8y + 16 - 16) - 14$

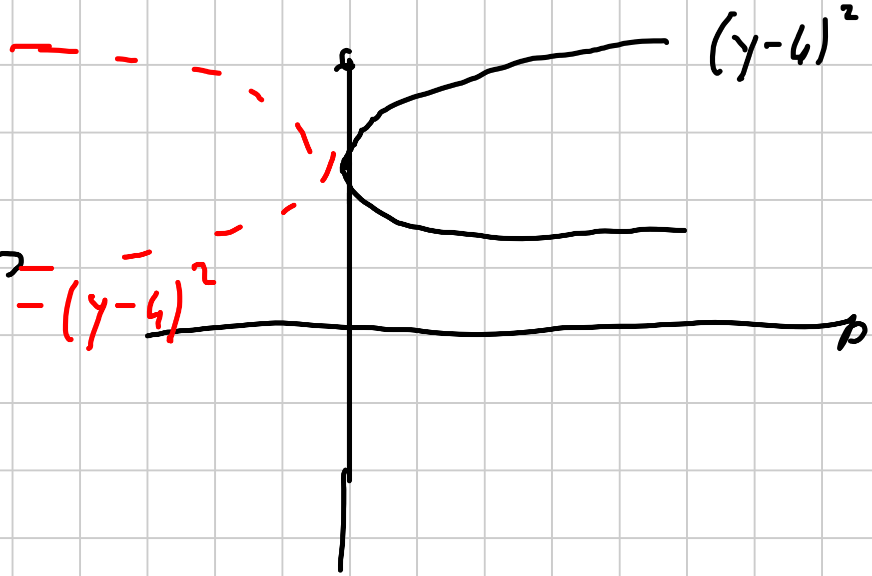
$x = -[(y^2 - 8y + 16) - 16] - 14$

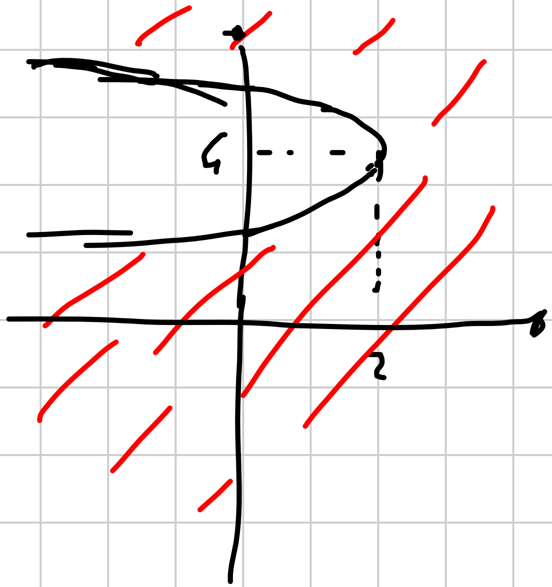
$x = -(+y^2 - 8y + 16) + 16 - 14$

$x = -(y-4)^2 + 2$

$V(2, 4)$







$x > -y^2 + 8y - 16$ $\quad (0,0)$

$0 > -16$ vera

Es:

$\mathcal{E}: \dfrac{(x-x_0)^2}{a^2} + \dfrac{(y-y_0)^2}{b^2} = 1$

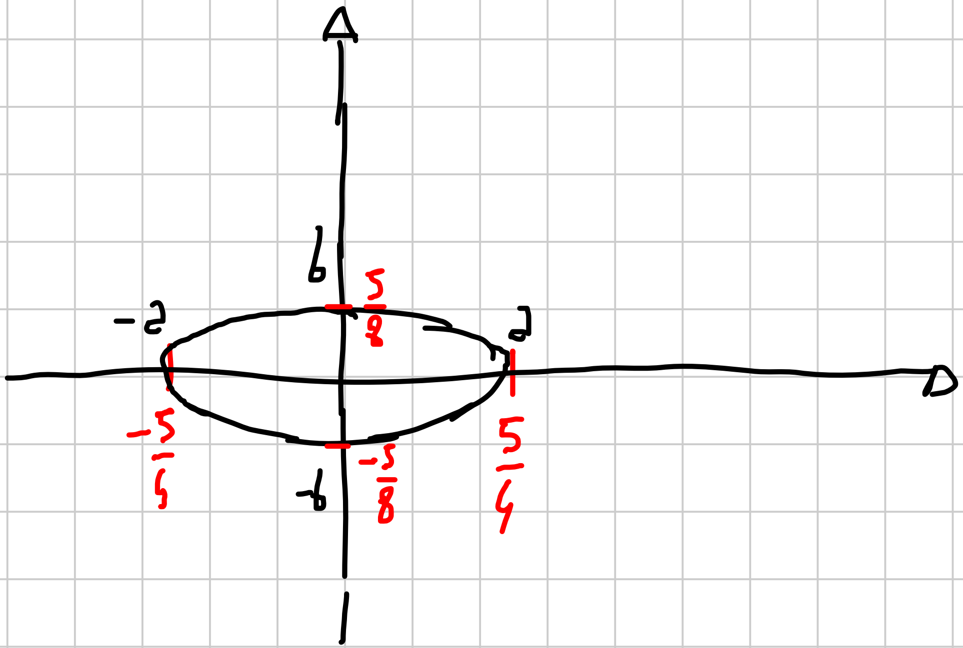
$C(x_0, y_0)$

$$\boxed{\frac{1x^2}{a^2} + \frac{1y^2}{b^2} = 1}$$

$16x^2 + 64y^2 = 25$

$$\frac{16x^2}{25} + \frac{64y^2}{25} = 1$$

$$\frac{x^2}{\frac{25}{16}} + \frac{y^2}{\frac{25}{64}} = 1$$



$a^2 = \frac{25}{16}$ $\quad$ $a = \frac{5}{4}$

$b^2 = \frac{25}{64}$ $\quad$ $b = \frac{5}{8}$

ES: $x^2 + 4y^2 - 4x + 8y - 17 = 0$

$$x^2 - 4x + 4y^2 + 8y - 17 = 0$$

$$x^2 - 4x + 4(y^2 + 2y) - 17 = 0$$

$$\underline{x^2 - 4x + 4} - 4 + 4(\underline{y^2 + 2y + 1} - 1) - 17 = 0$$

$$(x-2)^2 - 4 + 4(y^2 + 2y + 1) + 4\cdot(-1) - 17 = 0$$

$$(x-2)^2 - 4 + 4(y+1)^2 - 4 - 17 = 0$$

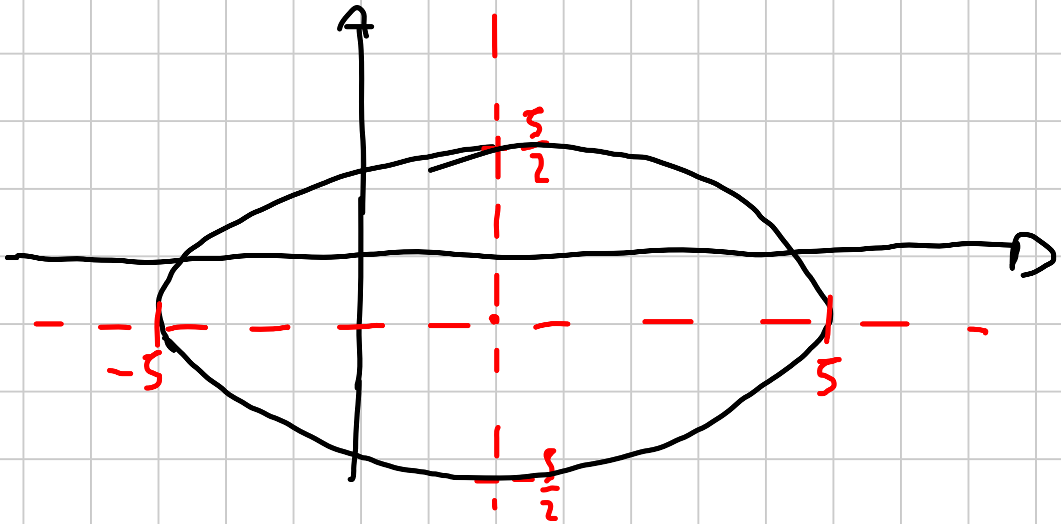
$$(x-2)^2 + 4(y+1)^2 = 25$$

$$\frac{(x-2)^2}{25} + \frac{4(y+1)^2}{25} = 1$$

$$\frac{(x-2)^2}{25} + \frac{(y+1)^2}{\frac{25}{4}} = 1$$

$C(2,-1)$

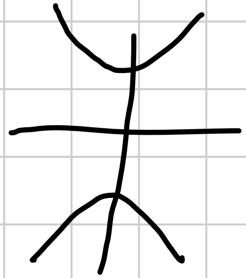
$a = 5 \quad b = \frac{5}{2}$



$$J_1: \frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$$

ES:

$$J_2: \frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = -1$$

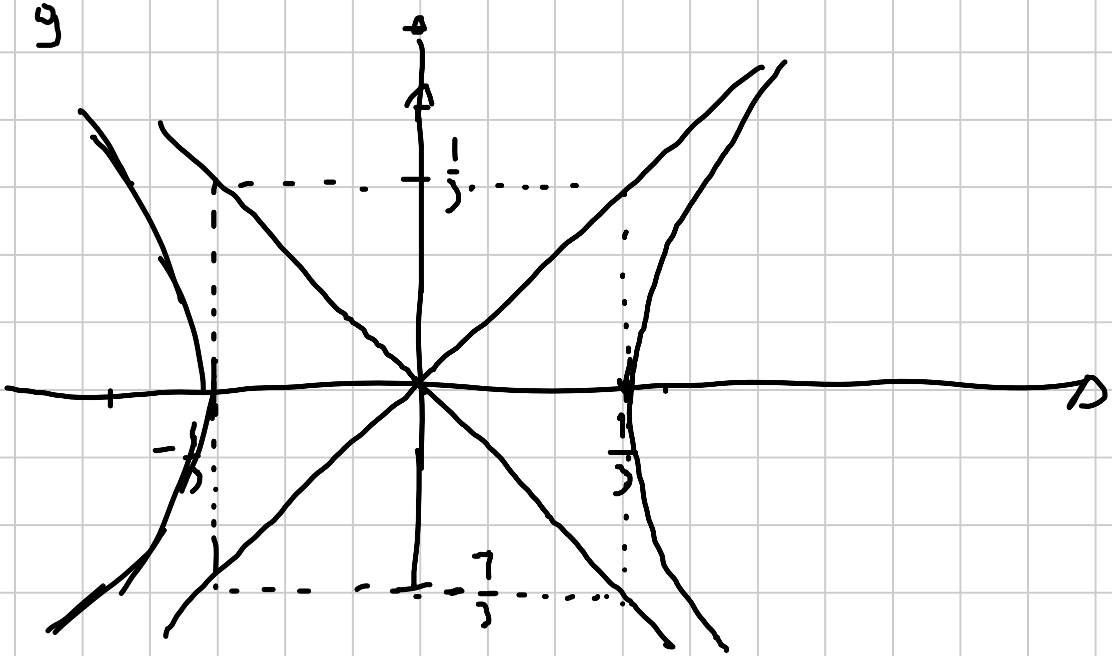


$$9x^2 - 9y^2 = 1$$

$$\frac{x^2}{\frac{1}{9}} - \frac{y^2}{\frac{1}{9}} = 1$$

$a = \frac{1}{3}$

$b = \frac{1}{3}$



$$x^2 - 25y^2 = -25$$

$$x^2 - 25y^2 < -25$$

$$\frac{x^2}{25} - y^2 = -1$$

$\exists$ con fuochi asse y

$0 < -25$

$a = 5$

$b = 1$