09/12/14

•) Dati i vettori

$$v^1 = (1, -2, -1) \qquad v^2 = (-1, 1, -2)$$

$$v^3 = (-2, 1, -1)$$

si trovi $\underset{(v^1)^T}{v} = \underbrace{2v^1 - v^2 + 2v^3}$

$$\vec{v} = \underbrace{2 \cdot \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}} - \underbrace{1 \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}} + \underbrace{2 \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}}$$

$v^1 \quad v_1$

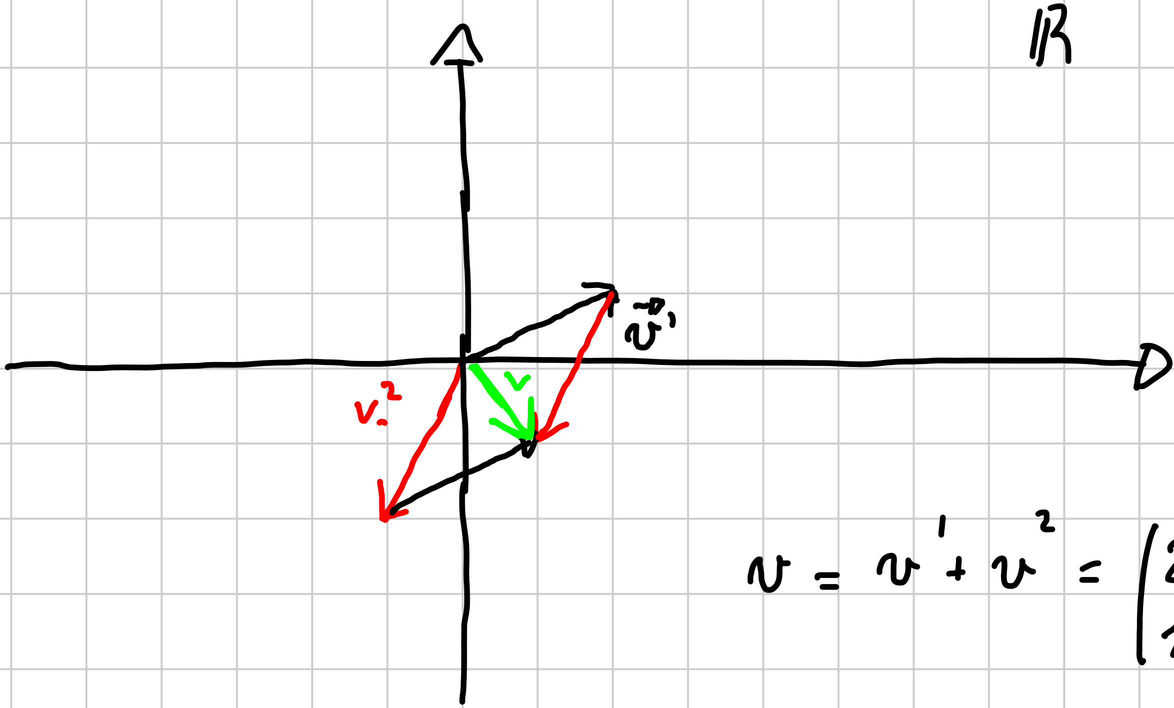
v_1^1

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ +2 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ -2 \end{pmatrix} = \vec{v}$$

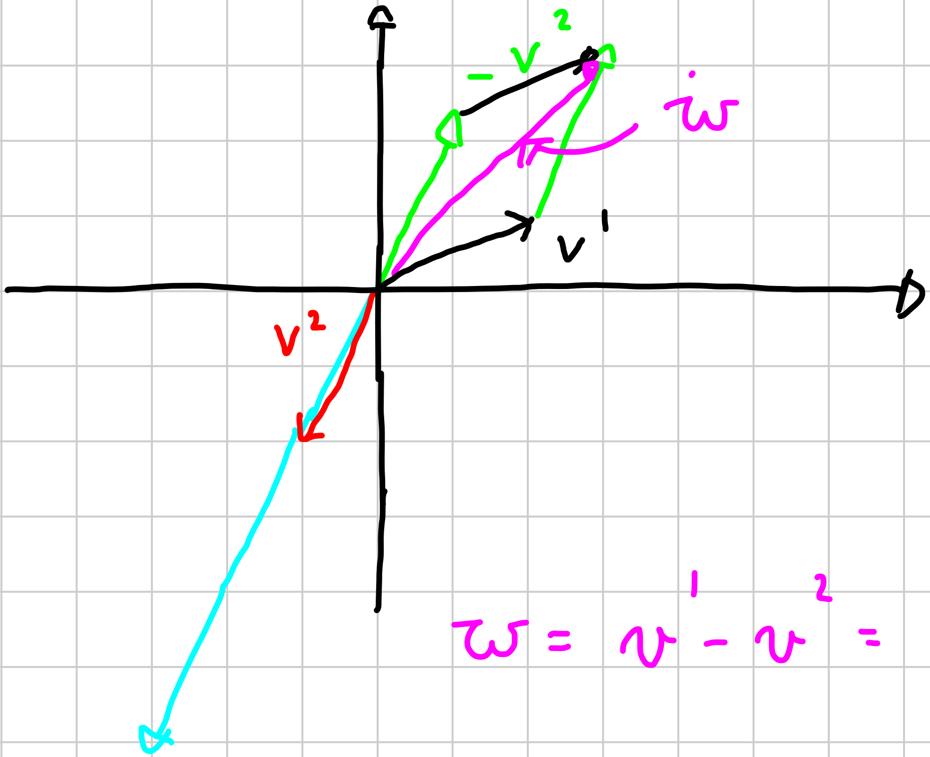
•) $v^1 = (2, 1) \qquad v^2 = (-1, -2) \qquad \vec{v} = v^1 + v^2$

$\mathbb{R}^2 \qquad \vec{w} = v^1 - v^2$



$$\vec{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

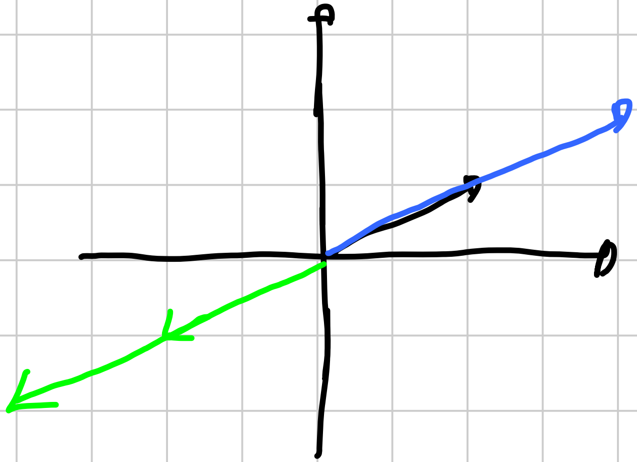
$$v = v^1 + v^2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$



$$v^1 - v^2 = \boxed{v^1 + (-v^2)}$$

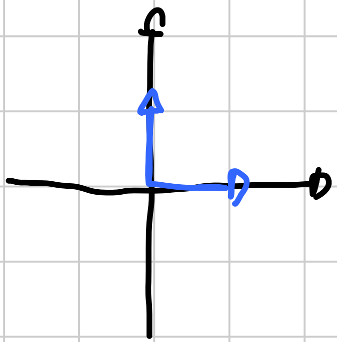
$$w = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

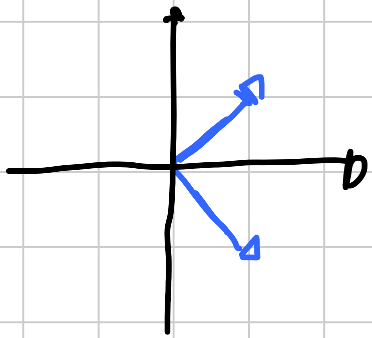
$$w = v^1 - v^2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$



$\cdot)\quad v^1 = (1, k, k-1) \qquad v^2 = (k, -k, 2)$

$? = k$ t.c. v^1 e v^2 ortogonali





ortogonali se $v^1 \cdot (v^2)^t$

riga 1×3 colonne 3×1

$$\underset{1 \times 3}{(1, k, k-1)} * \underset{3 \times 1}{\begin{pmatrix} k \\ -k \\ 2 \end{pmatrix}} = \quad 1 \times 1$$

$1 \cdot K + k(-k) + (k-1) \cdot 2$

$$\underset{3\times 4}{A} \cdot \underset{4\times 2}{B} = \underset{3\times 2}{C}$$

$$\underset{1\times 5}{v} \cdot \underset{5\times 3}{A} = \underset{1\times 3}{\underline{0}}$$

$$k - k^2 + 2k - 2 =$$

$$= -k^2 + 3k - 2$$

$$-k^2 + 3k - 2 = 0$$

$$-(k^2 - 3k + 2) = 0$$

$$-(k-2)(k-1) = 0$$

$$\boxed{k = 1 \lor k = 2}$$

Prove: $k = 1$ $\quad v^1 = (1, 1, 0) \quad v^2 = (1, -1, 2)$

$$v^1 \cdot (v^2)^T = (1, 1, 0) \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 1 \cdot 1 + 1 \cdot (-1) + 0 \cdot 2 = 0$$

$k = 2$ uguale

∘) $v^1 = (-1, 1)$ $\quad v^2 = (1, 1)$ sono l. i.

l i $\quad \boxed{a v^1 + b v^2 = 0} \quad a = b = 0$

$$a \begin{pmatrix} -1 \\ 1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -a + b = 0 \\ a + b = 0 \end{cases} \quad \begin{cases} b = a \\ a + a = 0 \end{cases} \quad \begin{cases} b = a \\ \frac{2a}{2} = \frac{0}{2} \end{cases} \quad \begin{cases} b = 0 \\ a = 0 \end{cases}$$

sono l. i.

Altro modo: $A = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$

$\det A \neq 0 \quad r(A) = \max$

$$\det \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} = -1 - 1 = -2 \neq 0 \Rightarrow r(A) = 2$$

$\Rightarrow v^1$ e v^2 l. i.

·) $v^1 = (1, -1, 0) \quad v^2 = (-1, 0, 1)$

$v^3 = (0, 1, 1)$

? = l. i.

$$\begin{array}{cccc} & 1,1 & 1,2 & \\ A = & \left(\begin{matrix} 1 & -1 \\ -1 & 0 \\ 0 & 1 \end{matrix}\right. & & \left.\begin{matrix} 0 \\ 1 \\ 1 \end{matrix}\right) \end{array}$$

$$\det(A) = 1 \cdot (-1)^{1+1} \cdot \det\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} + (-1)(-1)^{1+2} \cdot \det\begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\cancel{+\ 0 \cdot (-1)^{1+3} \cdot \det\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$= 1 \cdot (-1) + 1 \cdot (-1) = -1 - 1 = -2$$

$$\det A = -2 \neq 0 \Rightarrow r(A) = 3 \qquad \Rightarrow \text{l. i.}$$

$$a v^1 + b v^2 + c v^3 = 0 \qquad a = b = c = 0 \Rightarrow \text{l. i.}$$

$$a \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + b \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + c \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0$$

$$\begin{cases} a - b = 0 \\ -a + c = 0 \\ b + c = 0 \end{cases} \qquad a = b = c = 0 \qquad \text{l. i.}$$

•) $v^1 = (0, 1, -1) \quad v^2 = (-1, 0, 1) \quad v^3 = (1, -1, 0)$

? = l.i

$$A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \qquad \det \neq 0 \Rightarrow r = 2 \qquad \begin{pmatrix} 0 & -1 \\ 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$\det A$ se $\det A \neq 0 \Rightarrow r(A) = 3$

$$\det A = \cancel{0 \cdot \det\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}} - 1(-1)^3 \det\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} + 1(-1)^4 \cdot \det\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$= 1 \cdot (-1) + 1 \cdot (1) = 0$$

$$\det A = 0 \Rightarrow r(A) < 3$$

~~so~~ $r(A) = 2$ i vettori sono l. d.

so che $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ l. i.

$$\left\{ \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\} \qquad \dim S = 2$$

·) Scrivere v^3 come c.l. di v^1 e v^2?

$$a v^1 + b v^2 = v^3$$

$$a \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\begin{cases} -b = 1 \\ a = -1 \\ -a + b = 0 \end{cases} \qquad \begin{cases} a = -1 \\ b = -1 \\ -(-1) - 1 = 0 \text{ ok;} \end{cases}$$

$$av^1 + bv^2 + cv^3 = 0 \qquad a = b = c = 0$$

$$a\begin{pmatrix}0\\1\\-1\end{pmatrix} + b\begin{pmatrix}-1\\0\\1\end{pmatrix} + c\begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\begin{cases}-b+c=0\\ a-c=0\\ -a+b=0\end{cases} \qquad \begin{cases}-b+a=0\\ c=a\\ -a+b=0\end{cases} \qquad \begin{cases}b=a\\ c=a\\ -a+a=0\end{cases} \qquad \begin{cases}b=a\\ c=a\\ 0a=0\end{cases}$$

ind.

$$0a = 0$$

$$\begin{cases} a = 1 \\ b = 1 \\ c = 1 \end{cases} \qquad \begin{cases} a = 2 \\ b = 2 \\ c = 2 \end{cases}$$

~~l.in~~ $v^1 \; v^2 \; v^3$ l.d

•) Dire se $u = (1, 1, 1) \in S$

$$a v^1 + b v^2 = u$$

$$a \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + b \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} -b = 1 \\ a = 1 \\ -a+b = 1 \end{cases} \qquad \begin{cases} b = -1 \\ a = 1 \\ -1-1 = 1 \end{cases} \qquad \text{NO} \quad u \notin S$$

SISTEMA LINEARE $\qquad A\vec{x} = b$

$$\begin{cases} x - y + 2z = 1 \\ 2x + y - z = -1 \end{cases} \qquad \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$r(A) = r(A|b) \Rightarrow$ ha soluzione

n° incognite $- r(A) = 0$ c'è una sola sol.

altrimenti infinite

Calcolo $r(A)$ $\quad \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 2 & 1 & -1 \end{array}\right] \Rightarrow r(A) = 2$

$$r(A|b) = \left[\begin{array}{cc|cc} 1 & -1 & 2 & 1 \\ 2 & 1 & -1 & -1 \end{array}\right] \qquad r(A|b) = 2$$

$r(A) = r(A|b) = 2 \Rightarrow$ il sistema è possibile

n° incognite $- r(A) = 3 - 2 = 1 \Rightarrow \infty$ soluzioni

n° incognite $- r(A) = 1 =$ dim. spazio delle soluzioni

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 1 & -1 \end{array}\right]^{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{cases} x + y + 2z = 1 \\ 2x + y - z = -1 \end{cases} \qquad \begin{cases} x + y = 1 - 2z \\ 2x + y = -1 + z \end{cases}$$

$$\boxed{\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 - 2z \\ -1 + z \end{pmatrix}$$

$$y = 1 - 2z + 2 - 3z$$

$$\begin{cases} y = 1 - 2z - x \\ 2x + 1 - 2z - x = -1 + z \end{cases} \qquad \begin{cases} * \\ x = -2 + 3z \end{cases} \qquad \begin{cases} \\ x = -2 + 3z \end{cases}$$

sol $\begin{cases} x = -2 + 3z \\ y = 3 - 5z \\ z \end{cases}$

$$\underbrace{\begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}} + z \underbrace{\begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}}$$

s. particolare sol. sistema
ovvero enunciato

$$\left\{ \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix} \right\}.$$