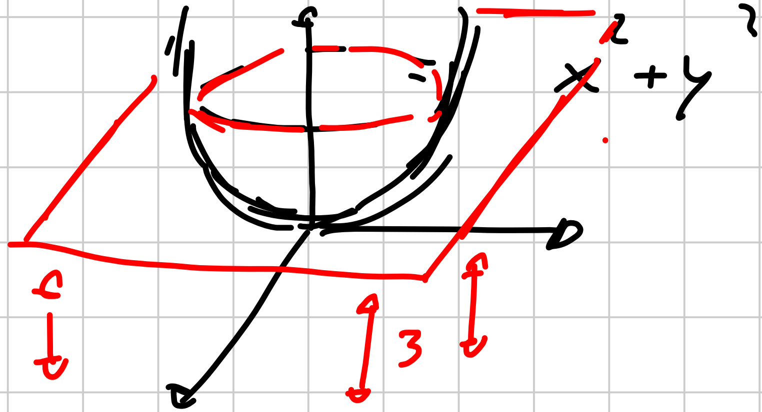
•) $f(x,y) = \sqrt{x(1-y^2)}$



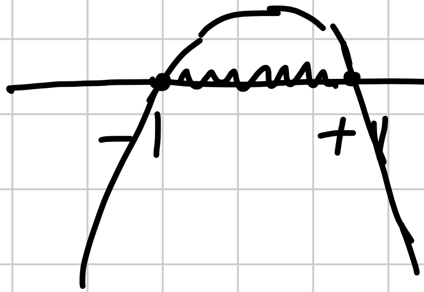
$$\underbrace{x}\,\underbrace{(1-y^2)} \geq 0$$

a) $\begin{cases} x \geq 0 \\ (1-y^2) \geq 0 \end{cases}$ $\cup$ b) $\begin{cases} x \leq 0 \\ 1-y^2 \leq 0 \end{cases}$

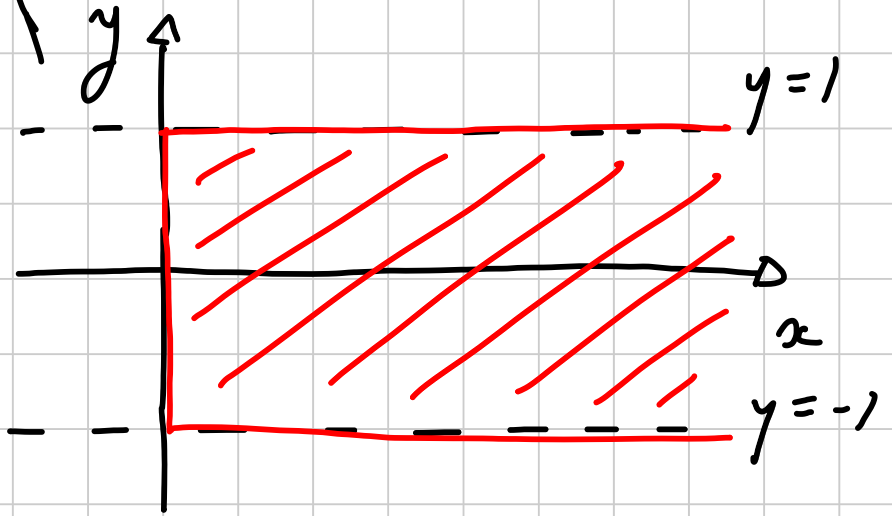
a) $\begin{cases} x \geq 0 \\ 1 - y^2 \geq 0 \end{cases}$

$1 - y^2 = 0$

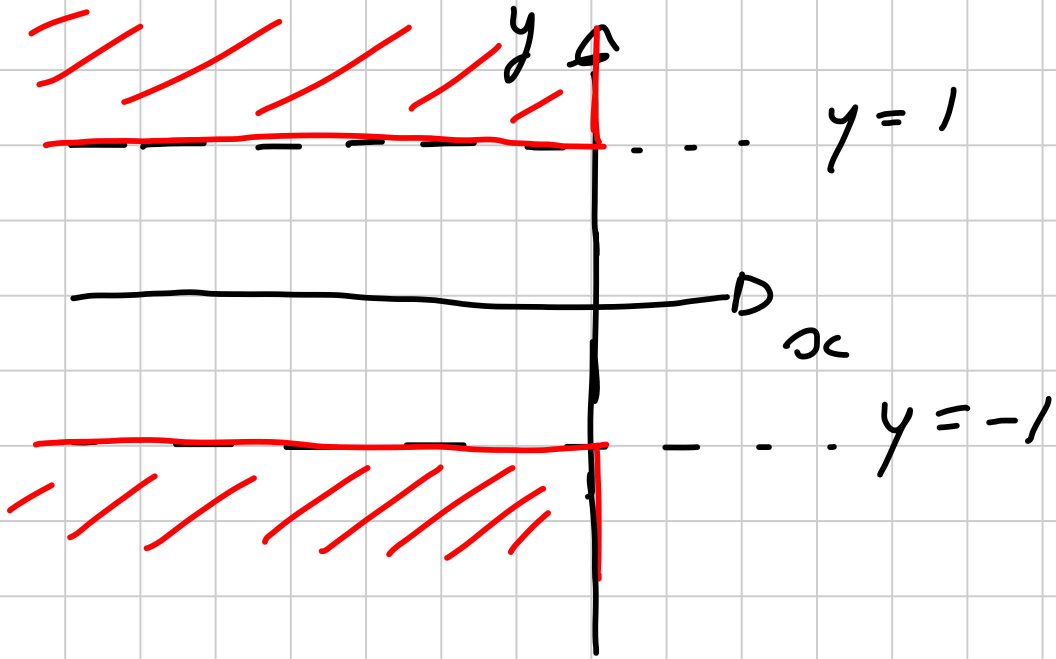
$y = \pm 1$



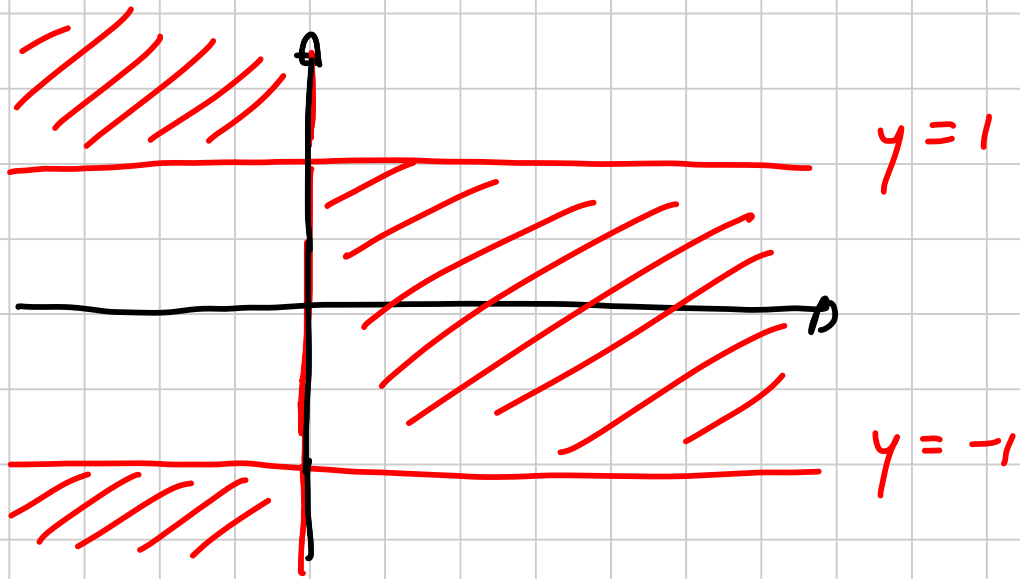
$\begin{cases} x \geq 0 \\ -1 \leq y \leq 1 \end{cases}$



b) $\begin{cases} x \leq 0 \\ 1 - y^2 \leq 0 \end{cases}$ $\quad$ $\begin{cases} x \leq 0 \\ y \leq -1 \ \vee \ y \geq 1 \end{cases}$



y
x
y = 1
y = -1



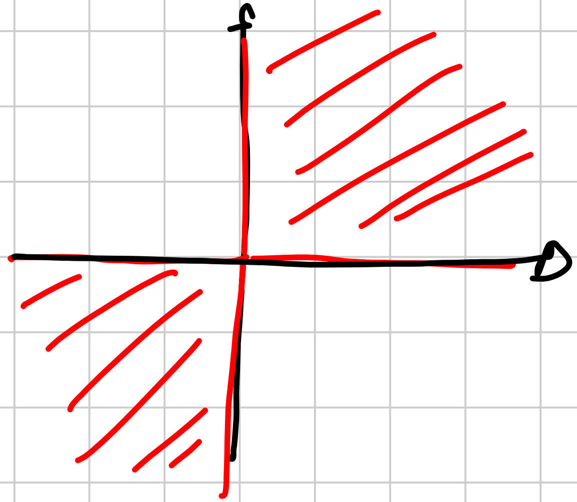
y = 1
y = -1

•) $f(x,y) = \sqrt{xy} + \sqrt{1-x^2-y^2}$

$$\begin{cases} xy \geq 0 & \text{a)} \\ 1-x^2-y^2 \geq 0 & \text{b)} \end{cases}$$

a) $xy \geq 0$

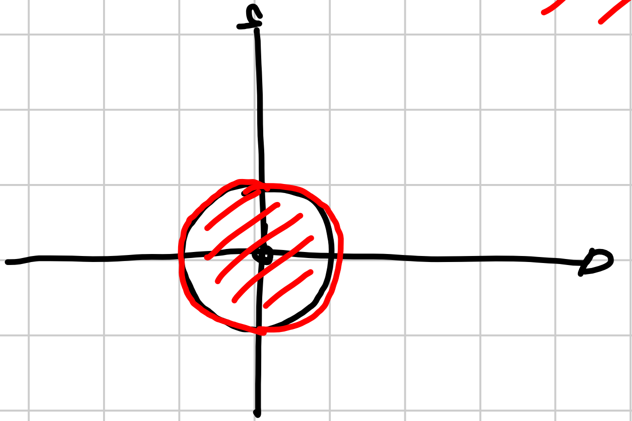
$$\begin{cases} x \geq 0 \\ y \geq 0 \end{cases} \quad \vee \quad \begin{cases} x \leq 0 \\ y \leq 0 \end{cases}$$



b) $1 - x^2 - y^2 \geq 0$

$-x^2 - y^2 \geq -1$

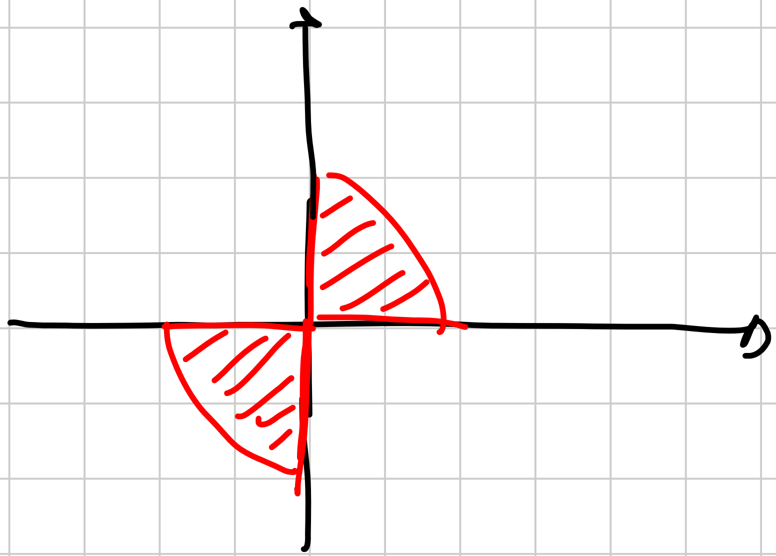
$x^2 + y^2 \leq 1$



$x^2 + y^2 = 1$

$O(0,0)$ $\boxed{x^2 + y^2 \leq 1}$

$0 \leq 1$ vero



- $f(x,y) = 1 - x^2 + y^2$ curva di livello 0

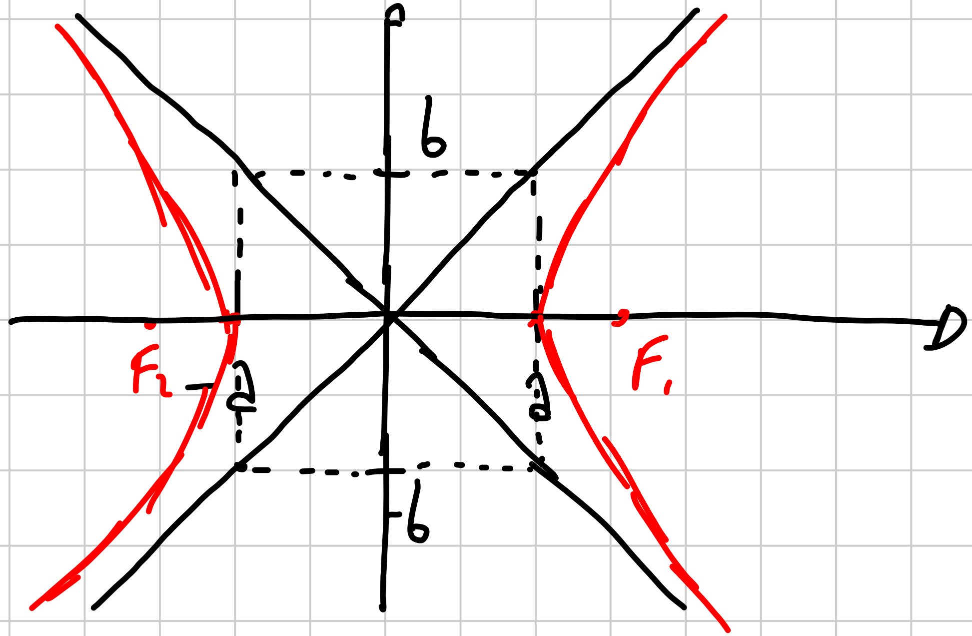
$f(x,y) = 0$

$1 - x^2 + y^2 = 0$

$-x^2 + y^2 = -1$

$x^2 - y^2 = 1$]

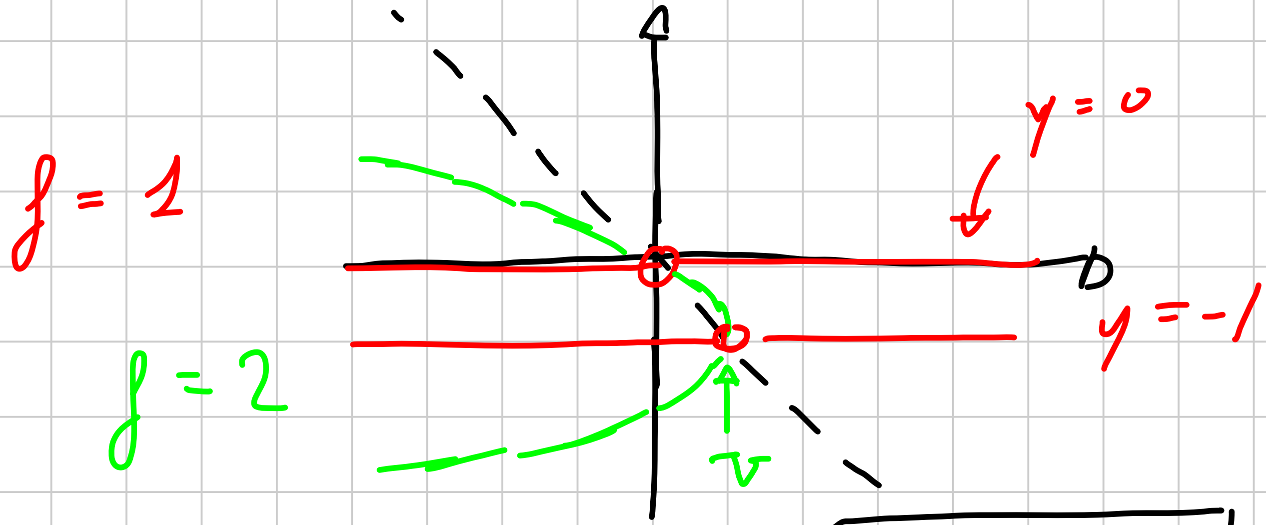
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \begin{array}{l} a = 1 \\ b = 1 \end{array}$$



•) $f(x,y) = \dfrac{x - y^2}{x + y}$?: livello 1 livello 2

D. di f $x + y \neq 0$

$y \neq -x$



$$\frac{x-y^2}{x+y} = 1 \rightarrow \cancel{x} - y^2 = \cancel{x} + y \rightarrow \boxed{0 = y^2 + y}$$

$$\rightarrow 0 = y(y+1) \rightarrow y = 0 \vee y = -1$$

$$\frac{x-y^2}{x+y} = 2 \rightarrow x - y^2 = 2(x+y) \rightarrow$$

$$x - y^2 = 2x + 2y \rightarrow x = -y^2 - 2y \quad P$$

$$x = -(y^2 + 2y)$$

$$x = -(y^2 + 2y + 1 - 1)$$

$$x = -(\underbrace{y^2 + 2y + 1}_{\text{quadrato}}) - (-1)$$

$$x = -(y+1)^2 + 1 \qquad V(1, -1)$$

•) $f(x,y) = x e^{xy}$ $\qquad x e^{2x}$

$$\frac{\partial f}{\partial x} = D[x] e^{2x} + x D[e^{2x}] = 1 e^{2x} + x \cdot 2 e^{2x}$$

$$D[e^{1x}] = 2 e^{2x} \qquad = e^{2x} + 2x e^{2x} \qquad = e^{xy} + xy e^{xy}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}[x] \cdot e^{xy} + x \cdot \frac{\partial}{\partial y}[e^{xy}]$$

$$= 0 \cdot e^{xy} + x \cdot x e^{xy} = x^2 e^{xy}$$

$$y \rightarrow xy = t \rightarrow e^t$$

$$\frac{d}{dy}[xy] \cdot \frac{d}{dt} e^t = x e^t = x e^{xy}$$

•) $f(x,y) = x e^{-x^2-y^2}$

$$\frac{d}{dx} = \frac{d}{dx}[x] e^{-x^2-y^2} + x \cdot \underbrace{\frac{d}{dx}[e^{-x^2-y^2}]}$$

$$= 1 e^{-x^2-y^2} + x(-2x) \cdot e^{-x^2-y^2}$$

$$= e^{-x^2-y^2} - 2x^2 e^{-x^2-y^2}$$

$$\frac{\partial}{\partial y} = \cancel{\frac{\partial}{\partial y}[x] e^{-x^2-y^2}} + x \frac{\partial}{\partial y}\left[e^{-x^2-y^2}\right] =$$

$$= x(-2y) e^{-x^2-y^2} = -2xy\, e^{-x^2-y^2}$$

•) $f(x,y) = \dfrac{2xy}{x^2+y^2}$

$$\frac{\partial f}{\partial x} = \frac{\frac{\partial}{\partial x}[2xy] \cdot (x^2+y^2) - 2xy \cdot \frac{\partial}{\partial x}[x^2+y^2]}{(x^2+y^2)^2}$$

$$= \frac{2y\,(x^2+y^2) - 2xy\,(2x)}{(x^2+y^2)^2} = \frac{2x^2y + 2y^3 - 4x^2y}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{\frac{\partial}{\partial y}[2xy] \cdot (x^2+y^2) - (2xy) \cdot \frac{\partial}{\partial y}[x^2+y^2]}{(x^2+y^2)^2}$$

$$= \frac{2x\,(x^2+y^2) - 2xy\,(2y)}{(x^2+y^2)^2}$$

Esame 22/01/14

ES. 1

$$f(x) = \frac{\ln^2 x}{x}$$

? = integrale indefinito

$$\int f(x)\,dx = \int \frac{\ln^2 x}{x}\,dx = \int \frac{1}{x} \cdot \ln^2 x\,dx = \int g'(x)\,g(x)^2$$

$$g(x) = \ln x$$
$$g'(x) = \frac{1}{x}$$

$$= \frac{g(x)^3}{3} = \frac{\ln^3 x}{3} + C$$

? = la media integrale di f nell'intervallo $[1, e]$

$$\frac{\int_1^e f(x)\,dx}{(e-1)} = \frac{\int_1^e \frac{\ln^2 x}{x}\,dx}{(e-1)} = \frac{\left[\frac{\ln^3 x}{3}\right]_1^e}{e-1} =$$

$$= \frac{\frac{\ln^3 e}{3} - \frac{\ln^3 1}{3}}{e-1} = \frac{\frac{1^3}{3} - \frac{0}{3}}{e-1} = \frac{\frac{1}{3}}{e-1} = \frac{1}{3(e-1)}$$

? = $\sum_{n=1}^{\infty} f(e^n)$ converge o diverge

$$f(x) = \frac{\ln^2 x}{x} \qquad f(e^n) = \frac{(\ln e^n)^2}{e^n} = \frac{(n)^2}{e^n} = \frac{n^2}{e^n}$$

$$\sum f(e^n) = \sum_{n=1}^{\infty} \frac{n^2}{e^n} = \sum a_n$$

$$\lim_{n \to +\infty} \frac{n^2}{e^n} = 0$$ può convergere

RAPPORTO

$$\lim_{n \to +\infty} \frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^2}{e^{n+1}}}{\frac{n^2}{e^n}} = \frac{(n+1)^2}{e^{n+1}} \cdot \frac{e^n}{n^2} =$$

$$\frac{(n+1)^2}{n^2 \cdot e}$$

$$\frac{e^n}{e^{n+1}} = \frac{1}{e^{\cancel{n}+1-\cancel{n}}} = \frac{1}{e}$$

$$\lim_{n\to+\infty} \frac{(n+1)^2}{n^2 e} = \lim_{n\to+\infty} \frac{\cancel{n^2}}{\cancel{n^2} e} = \frac{1}{e} < 1 \rightarrow \text{converge}$$

$$\sum \frac{n^2}{e^n}$$

$$\lim_{n\to+\infty} \frac{\frac{n^2}{e^n}}{\frac{1}{n^2}} = \lim_{n\to+\infty} \frac{n^4}{e^n} = 0 \rightarrow \text{converge}$$

$\frac{1}{n^2} \rightarrow$ converge

ES. 2

$v^1 = (0,-1,1)$ $v^2 = (1,2,-1)$ $v^3 = (-1,1,0)$

usando la definizione

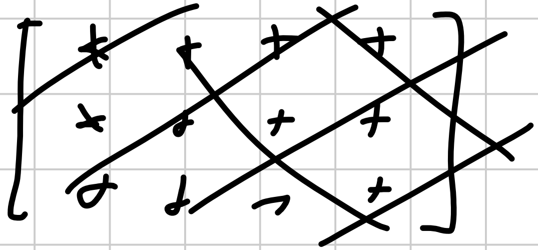
$$\left[\begin{array}{c} a = b = c = 0 \\ \text{l. i} \end{array}\right]$$

$$a v^1 + b v^2 + c v^3 = 0$$

$$a \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + b \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + c \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} b - c = 0 \\ -a + 2b + c = 0 \\ a - b = 0 \end{cases} \qquad \begin{cases} c = b \\ -b + 2b + b = 0 \\ a = b \end{cases} \qquad \begin{cases} c = 0 \\ b = 0 \\ a = 0 \end{cases} \qquad \text{l. i}$$

$$A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$



$$\det A \neq 0 \Rightarrow r(A) = \max$$

$$\begin{array}{c} \qquad\quad 2+1 \\ \qquad * \quad * \quad * \end{array}$$

$$\det \begin{pmatrix} 0 & 1 & -1 \\ -1 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix} = \cancel{0 \cdot \det \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}} - 1 \det \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$-1 \cdot \det \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} = 2$$

$$\det A \neq 0 \Rightarrow r(A) = 3$$

$$\begin{array}{ccccc} & v^1 & v^2 & v^3 & \\ \Bigg\{ & \begin{pmatrix}0\\-1\\1\end{pmatrix}, & \begin{pmatrix}1\\2\\-1\end{pmatrix}, & \begin{pmatrix}-1\\1\\0\end{pmatrix} & \Bigg\} \\ & + & + & + & \end{array}$$

$$a v^1 + b v^2 + c v^3 = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\overbrace{\underset{w^1}{\begin{pmatrix}0\\1\\2\\3\end{pmatrix}} \quad \underset{w^2}{\begin{pmatrix}1\\2\\3\\4\end{pmatrix}} \quad \underset{w^3}{\begin{pmatrix}2\\1\\3\\2\end{pmatrix}}}^{l.i.} \qquad \cancel{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}}$$

$$\cancel{a w^1} + b w^2 + c w^3 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

ES 3

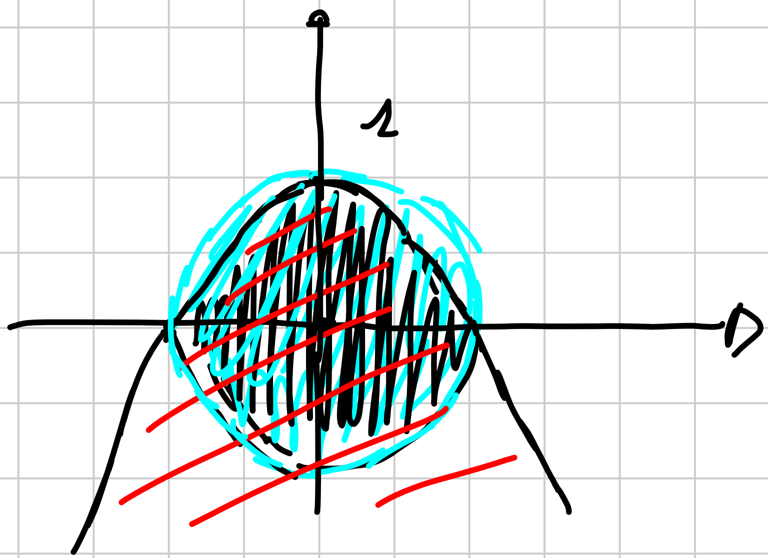
$$f(x,y) = \ln(1-x^2-y) - \ln(1-x^2-y^2)$$

$$\begin{cases} a) \quad 1-x^2-y > 0 \\ b) \quad 1-x^2-y^2 > 0 \end{cases}$$

a) $1-x^2-y > 0$

$x^2+y < 1$

$y < 1-x^2$



$$y = 1 - x^2$$

$O(0,0)$ $\qquad y < 1 - x^2$

$0 < 1$ vere

b) $1 - x^2 - y^2 > 0$

$$x^2 + y^2 < 1$$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

$$f(x,y) = \ln(1 - x^2 - y) - \ln(1 - x^2 - y^2)$$

$$\frac{\partial f}{\partial x} = -2x \cdot \frac{1}{1 - x^2 - y} - (-2x) \cdot \frac{1}{1 - x^2 - y^2}$$

$$\frac{\partial f}{\partial y} = \underbrace{-1 \cdot \frac{1}{(1-x^2-y)}} \underbrace{- (-2y) \cdot \cancel{\frac{1}{1-x^2-y^2}}}$$

? $O(0,0)$ punto stazionario ?