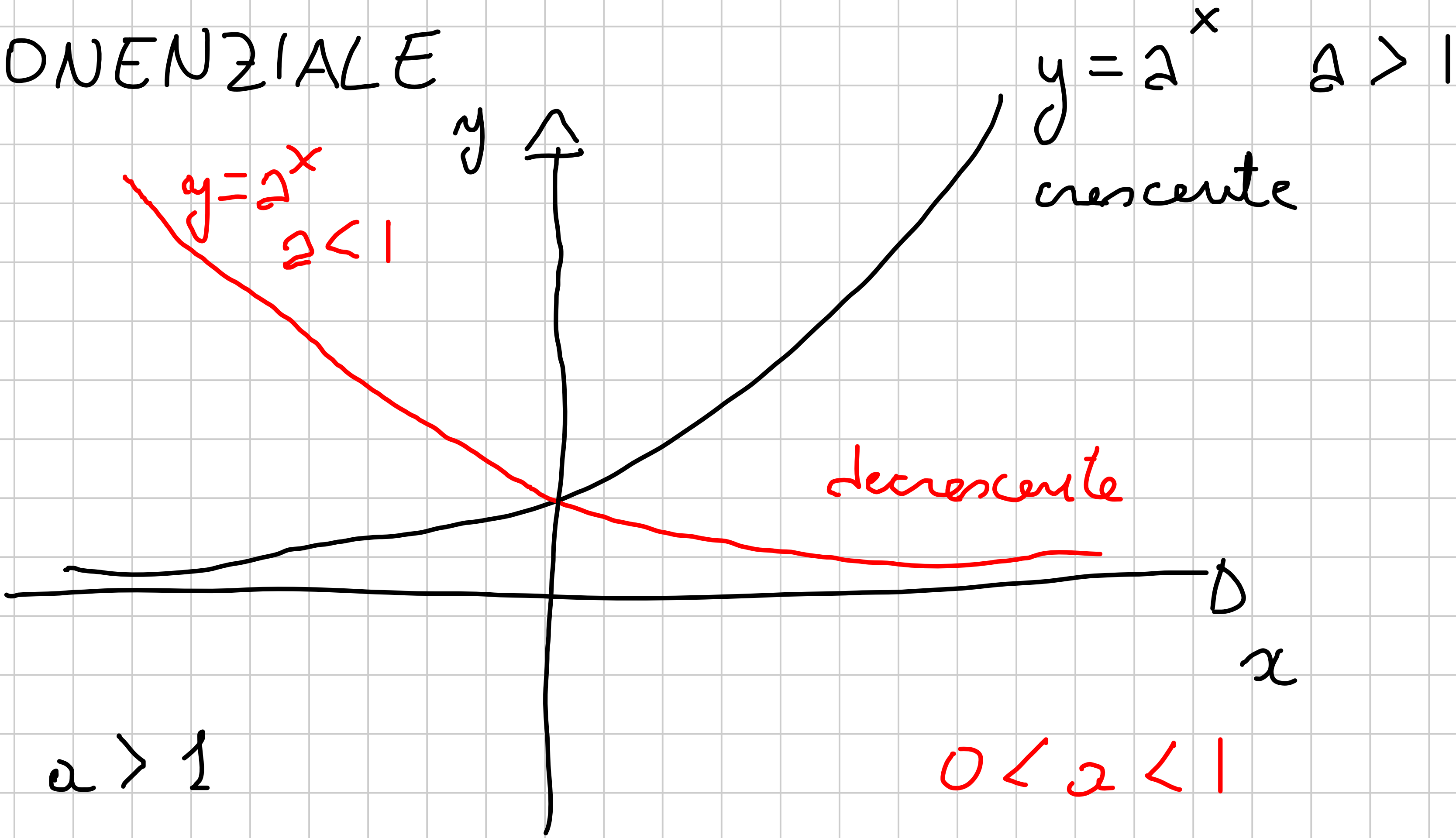
ESPONENZIALE



$a > 1$

$$\lim_{x \to +\infty} a^x = +\infty$$

$$\lim_{x \to -\infty} a^x = 0$$

$0 < a < 1$

$$\lim_{x \to +\infty} a^x = 0$$

$$\lim_{x \to -\infty} a^x = +\infty$$

$$e^{x^2+5} = e^{x^2} \cdot e^{5}$$

$$e^{x^2+5x-4} = e^{x^2} \cdot e^{5x} \cdot e^{-4} = \frac{e^{x^2} \cdot e^{5x}}{e^{4}}$$

$$\left(5^2\right)^{(x^2+1)} = 5^{2 \cdot (x^2+1)} = 5^{2x^2+2} = 5^{2x^2} \cdot 5^{2}$$

$$\left(5^2\right)^{7} = 5^{14}$$

1) $$e^{2x-1} = e^{\frac{1}{x}}$$

CE: $x \neq 0$

$$x\,(2x-1) = \frac{1}{\cancel{x}} \cdot \cancel{x}$$

$$x\,(2x-1) = 1$$

$$2x^2 - x - 1 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4}$$

$$x_1 = \frac{1+3}{4} = 1 \qquad x_2 = \frac{1-3}{4} = -\frac{1}{2}$$

$$x = -\frac{1}{2} \quad \vee \quad x = 1$$

2)

$$7^{\sqrt{x^2-1}} = 49$$

$$7^{\sqrt{x^2-1}} = 7^2$$

$$\left(\sqrt{x^2-1}\right)^2 = (2)^2$$

$$x^2 - 1 = 4$$

$$x^2 - 5 = 0$$

$$x^2 = 5$$

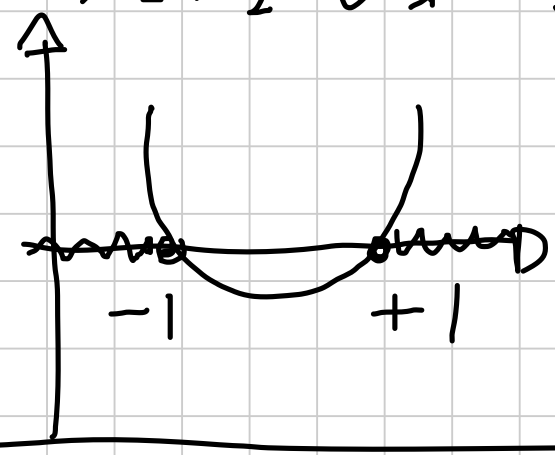
$$\begin{cases} x = \pm\sqrt{5} \\ x \leq -1 \vee x \geq 1 \end{cases} \qquad x = -\sqrt{5} \quad \vee \quad x = +\sqrt{5}$$

CE: $\boxed{x^2 - 1 \geq 0}$

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$$x = -1 \vee x = 1$$



$$\boxed{x \leq -1 \quad \vee \quad x \geq 1}$$

3) $5^{1+\sqrt{x}} + 5^{1-\sqrt{x}} = 10$ CE $\boxed{x \geq 0}$

$$\frac{5 \cdot 5^{\sqrt{x}}}{5} + \frac{5 \cdot 5^{-\sqrt{x}}}{5} = \frac{10}{5}$$

$$5^{\sqrt{x}} + 5^{-\sqrt{x}} = 2$$

$$5^{\sqrt{x}} \left(5^{\sqrt{x}} + \frac{1}{5^{\sqrt{x}}} \right) = 2 \cdot 5^{\sqrt{x}}$$

$$5^{2\sqrt{x}} + 1 = 2 \cdot 5^{\sqrt{x}}$$

$t = 5^{\sqrt{x}}$

$$\left(5^{\sqrt{x}}\right)^2 - 2 \cdot 5^{\sqrt{x}} + 1 = 0$$

$$t^2 - 2t + 1 = 0$$

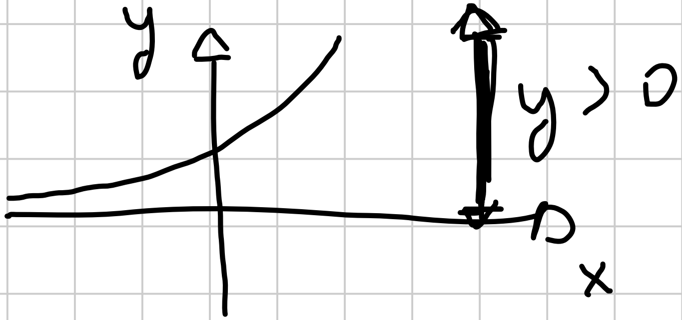
$$(t-1)^2 = 0$$

$$t = 1$$

$$5^{\sqrt{x}} = 1$$

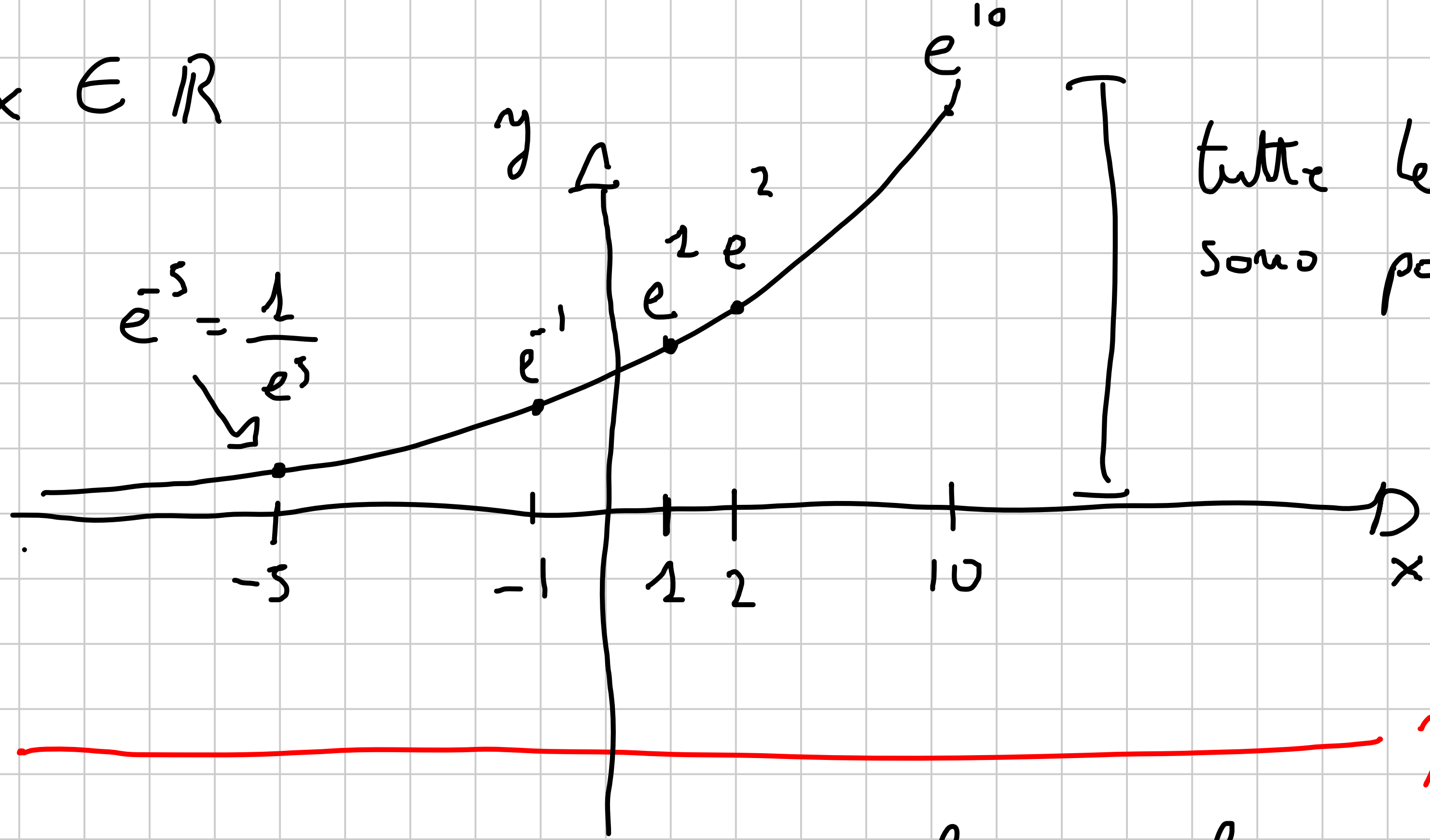
$$5^{\sqrt{x}} = 5^0 \longrightarrow \sqrt{x} = 0 \qquad \begin{cases} x = 0 \\ x \geq 0 \end{cases} \qquad x = 0$$

) $e^{7x-2} \neq -8$ $\qquad \cancel{-8 = e^{y}}$



$\cancel{y = \ln_e(-8)}$

$\forall x \in \mathbb{R}$



tutte le y sono positive

$y = -8$

$e^x > -8$

ln $\log_2$

$\ln_e$ $\ln_2$

) $\dfrac{e^{x+1} - e^x}{e^{5x} - 1} \leq 0$

CE $e^{5x} - 1 \neq 0$

$e^{5x} \neq 1$

$e^{5x} \neq e^0$

$5x \neq 0$

$\boxed{x \neq 0}$

$N \geq 0$

$e^{x+1} - e^{x} \geq 0$

$e^{x+1} \geq e^{x}$

$x + 1 \geq x$ poiché e^{x} è crescente

$x - x \geq -1$

$0x \geq -1$

$0 \geq -1$ $\forall x \in \mathbb{R}$

$D > 0$

$e^{5x} - 1 > 0$

$e^{5x} > 1$

$e^{5x} > e^{0}$

$5x > 0 \rightarrow x > 0$

		0	
N	$+$		$+$
D	$-$		$+$
$\frac{N}{D}$	$-$		$+$

$x < 0$

)

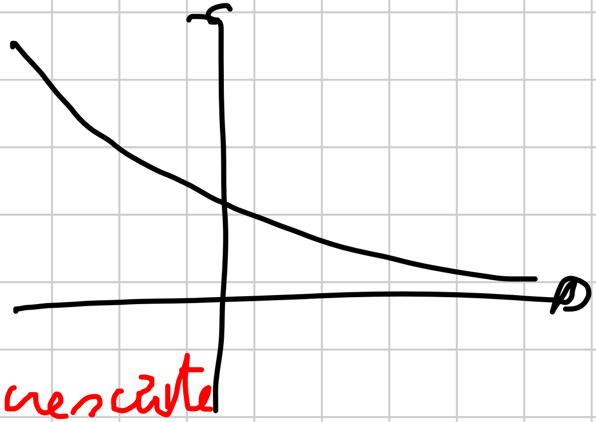
$$\left(\frac{1}{2}\right)^{x} - \left(\frac{1}{2}\right)^{-x} \geq 0$$

$$\left(\frac{1}{2}\right)^{x} \geq \left(\frac{1}{2}\right)^{-x}$$

$$x \leq -x$$

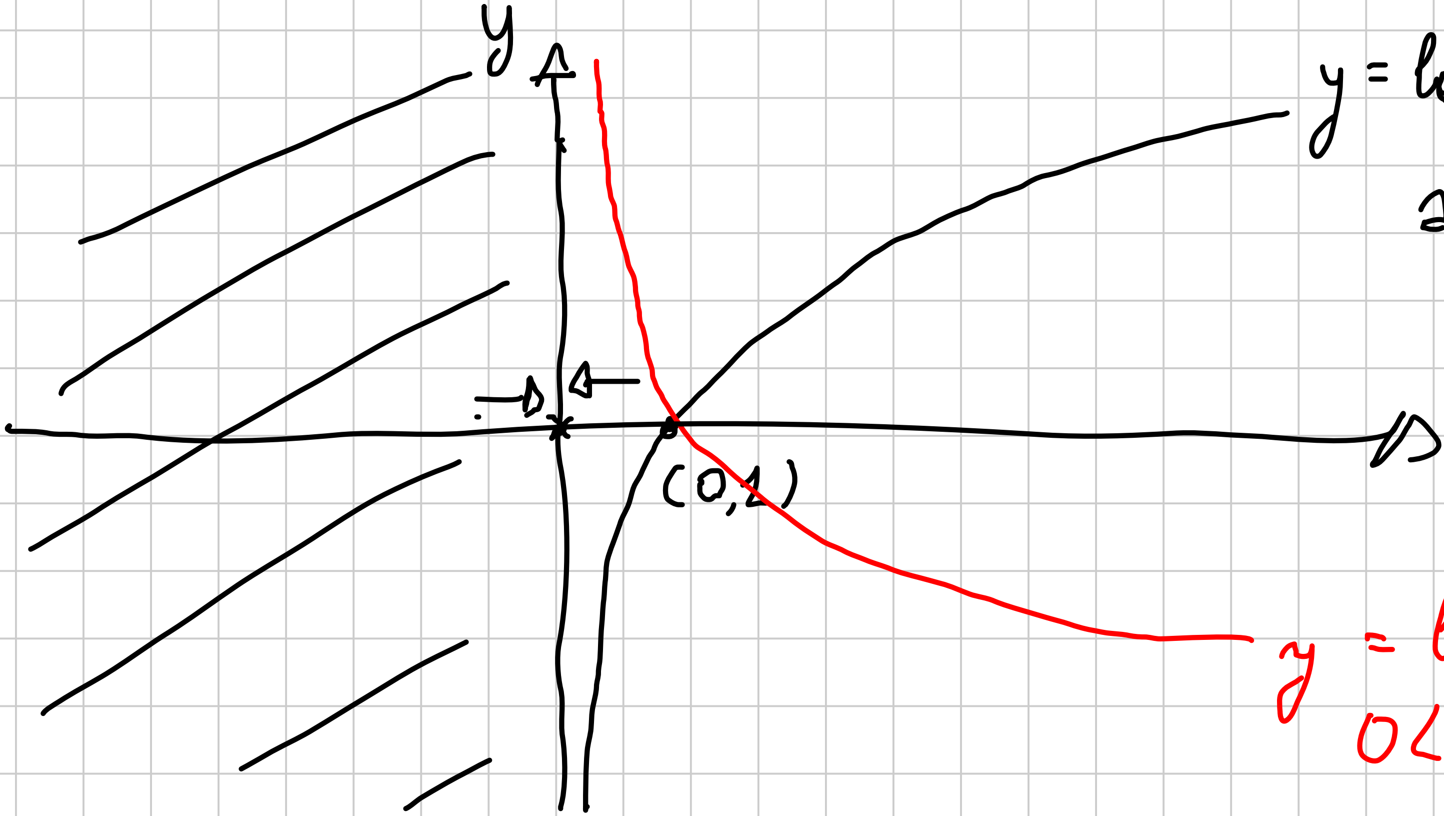
$$\frac{\cancel{2}x}{\cancel{2}} \leq \frac{0}{2}$$

$$x \leq 0$$



poiché decrescente

LOGARITMI



$y = \log_a x$

$a > 1$

$y = \ln_a x$

$0 < a < 1$

$a > 1$

$$\lim_{x \to +\infty} \log_a x = +\infty$$

$$\lim_{x \to 0^+} \ln_a x = -\infty$$

$$\lim_{x \to 0^-} \ln_a x = \nexists$$

$0 < a < 1$

$$\lim_{x \to +\infty} \log_a x = -\infty$$

$$\lim_{x \to 0^+} \ln_a x = +\infty$$

$$\ln_a x + \ln_a y = \ln_a (x \cdot y)$$

$$\ln_a x - \ln_a y = \ln_a \left(\frac{x}{y}\right)$$

$$\ln_a x = \frac{\ln_c x}{\ln_c a}$$

$$\boxed{x > 0 \wedge y > 0}$$

$$\ln_a x^n = n \cdot \ln_a x$$

$$\ln x^2 \neq 2 \ln x$$

$$x^2 > 0 \qquad x > 0$$

$$x \neq 0$$

D

$D\ ]-\infty; 0[\ \cup\ ]0; +\infty[$

$$5^{\sqrt{x}} \cdot 5^{\sqrt{x}} = 5^{\sqrt{x}+\sqrt{x}} = 5^{2\sqrt{x}}$$

$$2^{n} \cdot 2^{m} = 2^{n+m}$$

$$2^{3} \cdot 2^{3} = 2^{8}$$

)

$$\ln_3 (x-4) = 2$$

CE

$$x-4 > 0$$

$$\boxed{x > 4}$$

$$2 = \ln_3 y$$

$$y = 3^{2}$$

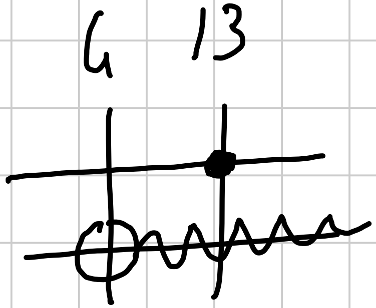
$$2 = \ln_3 3^{2}$$

$$\ln_3 (x-4) = \ln_3 3^{2}$$

$$x-4 = 9$$

$$x = +13$$

$$\begin{cases} x = 13 \\ x > 4 \end{cases}$$



$$x = 13$$

$\log_2 8 = 3$ $\qquad\qquad$ $\frac{1}{\sqrt[3]{2^2}} = 2^{?}$

$\log_2\left(\frac{1}{\sqrt[3]{2^2}}\right) = \log_2\left(\frac{1}{2^{\frac{2}{3}}}\right)$

$= \log_2\left(2^{-\frac{2}{3}}\right) = -\frac{2}{3}$

$\log_2\left(\frac{1}{8}\right) = \log_2\left(\frac{1}{2^3}\right) = \log_2 2^{-3} = -3$

) $\log_{\frac{1}{3}} x = -3$ $\qquad\qquad$ CE $x > 0$

$-3 = \log_{\frac{1}{3}} y \to \left(\frac{1}{3}\right)^{-3} = y \to \frac{1}{3^{-3}} = y$

$-3 = \log_{\frac{1}{3}}\left(\frac{1}{3}\right)^{-3}$

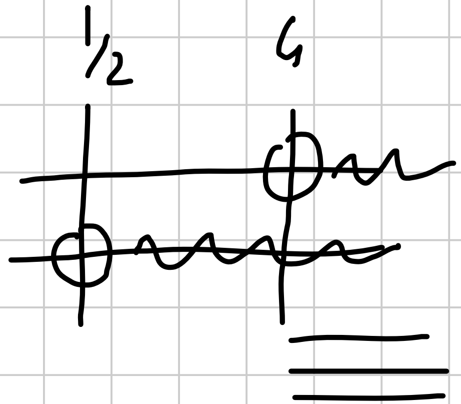
$$\log_{\frac{1}{3}} x = \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-3}$$

$$x = \left(\frac{1}{3}\right)^{-3} \longrightarrow x = \frac{1}{3^{-3}} \longrightarrow x = 3^{3} = 27$$

$$\begin{cases} x = 27 \\ x > 0 \end{cases} \qquad x = 27$$

) $\log_3 (\overbrace{x-4}^{+}) - \log_3 (\overbrace{2x-1}^{+}) = -1$

CE $\begin{cases} x-4 > 0 \\ 2x-1 > 0 \end{cases} \qquad \begin{cases} x > 4 \\ x > \frac{1}{2} \end{cases}$



$\boxed{x > 4}$

$$\log_3 \left(\frac{x-4}{2x-1}\right) = -1$$

$-1 = \log_3 y \rightarrow 3^{-1} = y$

$-1 = \log_3 3^{-1}$

$\log_3\left(\frac{x-4}{2x-1}\right) = \log_3(3^{-1})$

$(2x-1)\cdot\cancel{3}\ \frac{x-4}{2\cancel{x}-1} = \frac{1}{\cancel{3}}\cdot\cancel{3}\cdot(2x-1)$

$(2x-1)\cdot 3(x-4) = 3(2x-1)$ $\qquad x = 11$

da finire

) $\log_{\frac{1}{3}}(x^2-3) > 0$ $\qquad$ CE: $x^2-3>0$

$x^2-3=0$

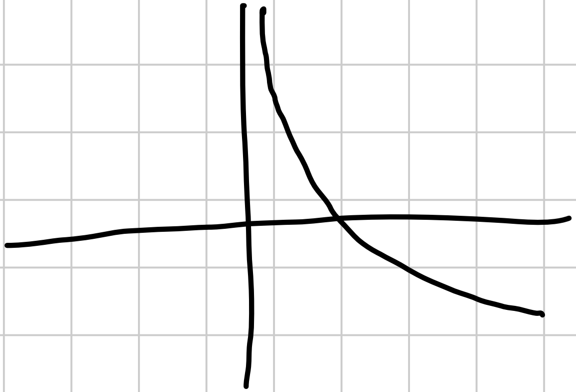
$x^2=3$

$x=\pm\sqrt{3}$

$0 = \log_{\frac{1}{3}} y$

$$y = \left(\frac{1}{3}\right)^{0}$$

$$\ln_{\frac{1}{3}}(x^2-3) > \ln_{\frac{1}{3}}\left(\frac{1}{3}\right)^{0}$$

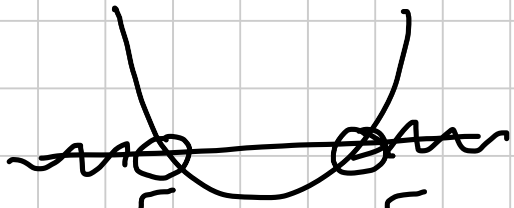


$$x^2 - 3 < 1$$

$$x^2 - 4 < 0$$

$$\cancel{\underbrace{(x+2)}\underbrace{(x-2)}} < 0$$

$$x^2 - 4 = 0$$

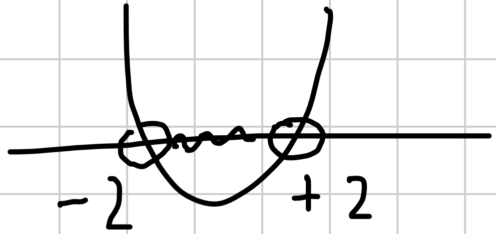


$-\sqrt{3}$ $\sqrt{3}$

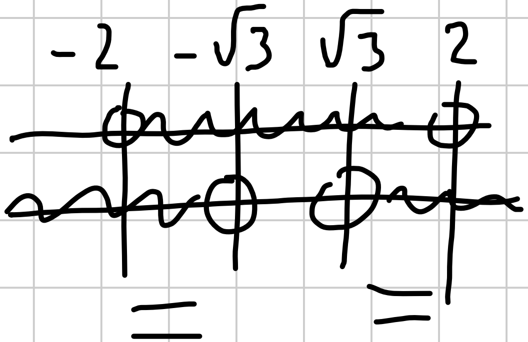
$$\boxed{x < -\sqrt{3} \vee x > \sqrt{3}}$$

$x^2 = 4$

$x = \pm \cancel{\sqrt{}}\,2$



$$\begin{cases} -2 < x < 2 \\ x < -\sqrt{3} \lor x > \sqrt{3} \end{cases}$$



$-2 < x < -\sqrt{3} \lor \sqrt{3} < x < 2$

F. COMPOSTA

$$f(x) = -\frac{3}{2}x + 7 \qquad g(x) = x^2 - 6x$$

$$f(g(x)) \qquad f \circ g \qquad g(f(x)) \qquad g \circ f$$

$$f(g(x)) = -\frac{3}{2}\left(x^2 - 6x\right) + 7$$

$$x \xrightarrow{g(x)} x^2 - 6x = t \xrightarrow{f(t)} -\frac{3}{2}t + 7 = -\frac{3}{2}\left(x^2 - 6x\right) + 7$$

$$g(f(x)) = \left(-\frac{3}{2}x + 7\right)^2 - 6\left(-\frac{3}{2}x + 7\right)$$

$$x \xrightarrow{f(x)} -\frac{3}{2}x + 7 = t \xrightarrow{g(t)} t^2 - 6t = \left(-\frac{3}{2}x + 7\right)^2 - 6\left(-\frac{3}{2}x + 7\right)$$

$$f(x) = x^2 + 1 \qquad g(x) = e^x$$

$$f(g(x)) = (e^x)^2 + 1 = e^{2x} + 1$$

$$x \xrightarrow{g(x)} e^x = z \xrightarrow{f(z)} z^2 + 1 = (e^x)^2 + 1 = e^{2x} + 1$$

$$g(f(x)) = e^{x^2+1}$$

$$x \xrightarrow{f(x)} x^2 + 1 = y \xrightarrow{g(y)} e^y = e^{x^2+1}$$

GRAFICO

CE: $x > -2$

$$f(x) = -\ln(x+2)$$