·) $\ln_2\left(x^2 - \frac{3}{4}\right) < -2$

CE: $x^2 - \frac{3}{4} > 0$

$x^2 - \frac{3}{4} = 0$

$-2 = \ln_2 2^{-2}$

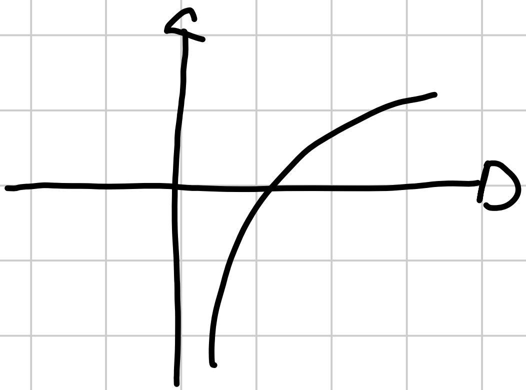
$-2 = \ln_2 y$

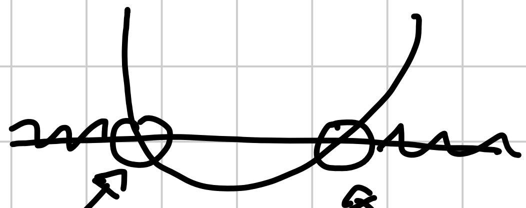
$x^2 = \frac{3}{4}$

⋮

$\ln_2\left(x^2 - \frac{3}{4}\right) < \ln_2 2^{-2}$

$x = \frac{\pm\sqrt{3}}{2}$





$-\frac{\sqrt{3}}{2}$ $\quad +\frac{\sqrt{3}}{2}$

$x^2 - \frac{3}{4} < \frac{1}{4}$

$x < -\frac{\sqrt{3}}{2} \ \vee \ x > \frac{\sqrt{3}}{2}$

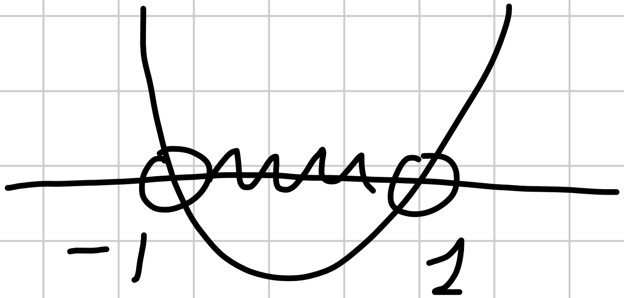
$$x^2 - \frac{3}{4} - \frac{1}{4} < 0$$

$$x^2 - 1 < 0$$

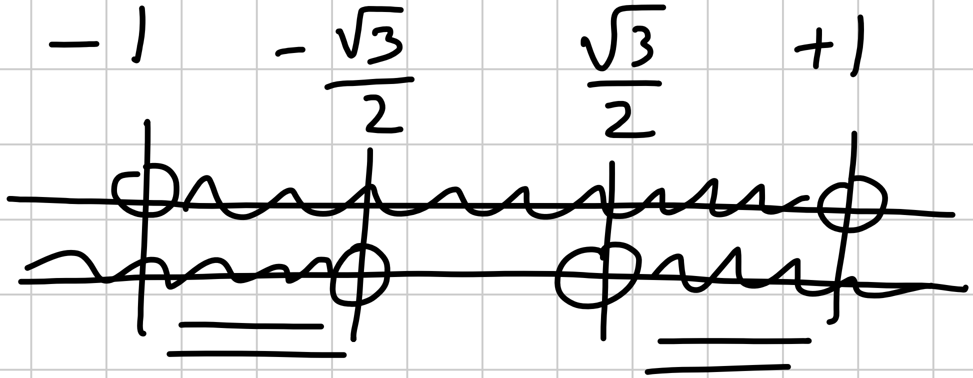
$$x^2 - 1 = 0$$

$$x = \pm 1$$

$$\begin{cases} -1 < x < 1 \\ x < -\frac{\sqrt{3}}{2} \;\vee\; x > \frac{\sqrt{3}}{2} \end{cases}$$



$$-1 < x < 1$$



$$-1 < x < -\frac{\sqrt{3}}{2} \;\vee\; \frac{\sqrt{3}}{2} < x < 1$$

•) $\ln\left(x - \frac{3}{2}\right) < -\ln x$

CE $\begin{cases} x - \frac{3}{2} > 0 \\ x > 0 \end{cases}$ $\begin{cases} x > \frac{3}{2} \\ x > 0 \end{cases}$ $\boxed{x > \frac{3}{2}}$

$$\ln x^n = n \ln x$$

$$-\ln x = \ln x^{-1}$$

$$\ln\left(x - \frac{3}{2}\right) < \ln x^{-1}$$

$$x - \frac{3}{2} < \frac{1}{x}$$

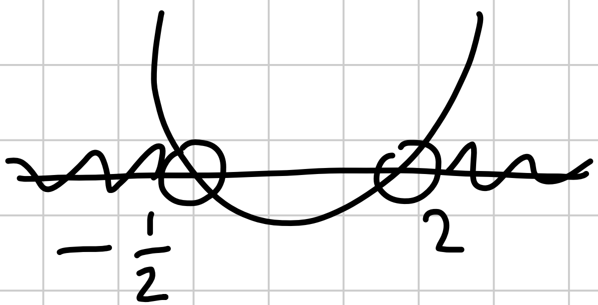
$$x - \frac{3}{2} - \frac{1}{x} < 0$$

$$\boxed{\frac{2x^2 - 3x - 2}{2x} < 0}$$

$N > 0 \qquad 2x^2 - 3x - 2 > 0$

$$2x^2 - 3x - 2 = 0$$

$$x_{1,2} = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm 5}{4} \quad \begin{cases} \frac{8}{4} = 2 \\ -\frac{2}{4} = -\frac{1}{2} \end{cases}$$

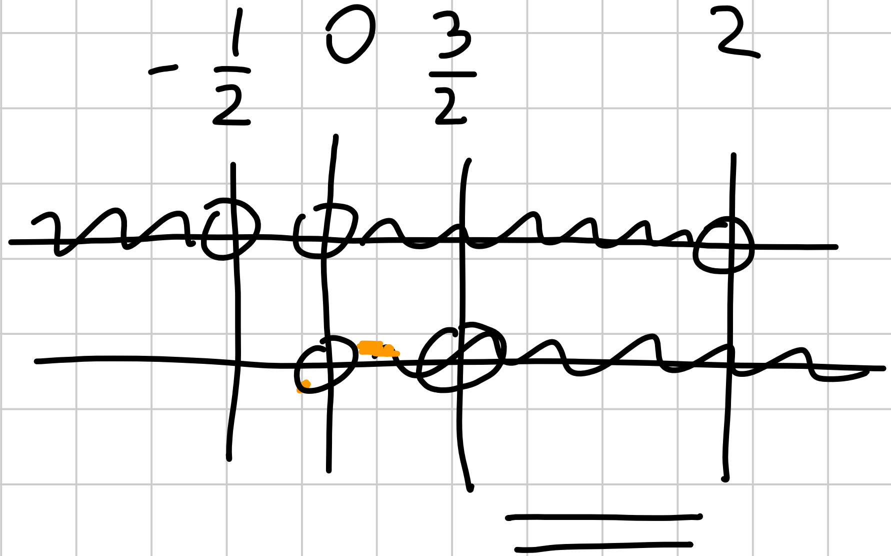


$$x < -\frac{1}{2} \ \vee \ x > 2$$

$D > 0 \qquad 2x > 0 \qquad x > 0$

	$-\frac{1}{2}$	0	2	
N	$+$	$-$	$-$	$+$
D	$-$	$-$	$+$	$+$
$\frac{N}{D}$	$-$	$+$	$-$	$+$

$$\begin{cases} x < -\frac{1}{2} \quad \vee \quad 0 < x < 2 \\ x > \frac{3}{2} \end{cases}$$



$$-\frac{3}{2} < x < 2$$

•) $2\ln x - \ln^2 x \geq \frac{1}{4}$ CE $\boxed{x > 0}$

$t = \ln x$

$2t - t^2 \geq \frac{1}{4}$

$\boxed{-t^2 + 2t - \frac{1}{4} \geq 0}$

$-t^2 + 2t - \frac{1}{4} = 0$

$-4t^2 + 8t - 1 = 0$

$t_{1,2} = \frac{-8 \pm \sqrt{64 - 16}}{-8} = \frac{-8 \pm \sqrt{48}}{-8}$

48	2
24	2
12	2
6	2
3	3
1	

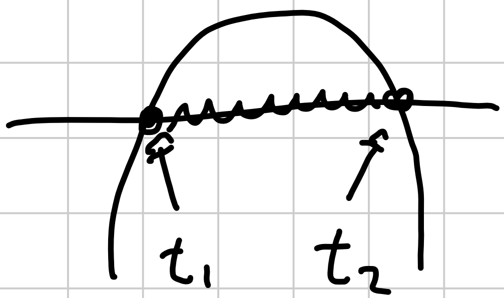
$$48 = 2^4 \cdot 3$$

$$\sqrt{48} = \sqrt{2^4 \cdot 3} = \sqrt{2^2 \cdot 2^2 \cdot 3} = 4\sqrt{3}$$

$$t_{1,2} = \frac{-8 \pm 4\sqrt{3}}{-8} = \frac{\cancel{4}(-2 \pm \sqrt{3})}{-\cancel{8}_2} = \frac{-2 \pm \sqrt{3}}{-2}$$

$$t_1 = \frac{-2+\sqrt{3}}{-2} = \frac{2-\sqrt{3}}{2}$$

$$t_2 = \frac{-2-\sqrt{3}}{-2} = \frac{2+\sqrt{3}}{2}$$



$$\ln x = \frac{2-\sqrt{3}}{2} \quad \vee \quad \ln x = \frac{2+\sqrt{3}}{2}$$

$$\frac{2-\sqrt{3}}{2} = \ln_e e^{\frac{2-\sqrt{3}}{2}}$$

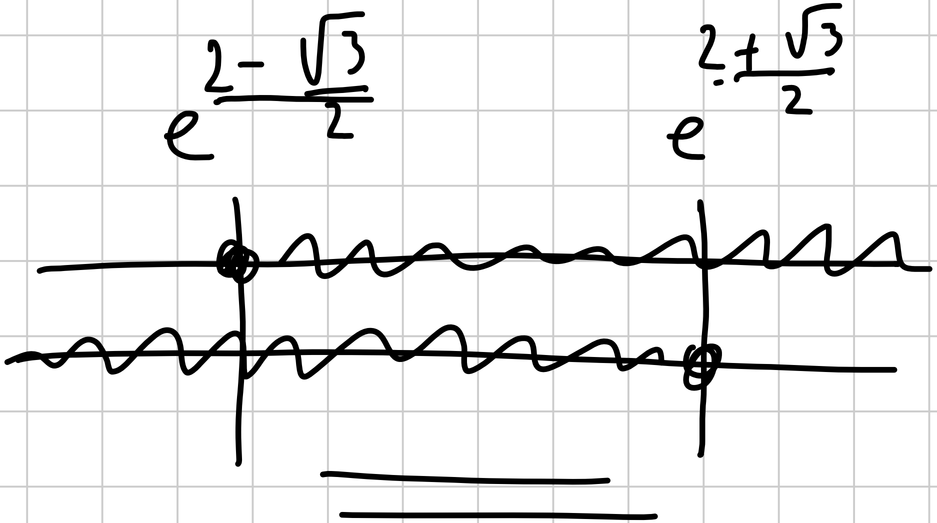
$$\ln x = \ln e^{\frac{2-\sqrt{3}}{2}}$$

$$x =$$

$$\frac{2-\sqrt{3}}{2} \leq t \leq \frac{2+\sqrt{3}}{2}$$

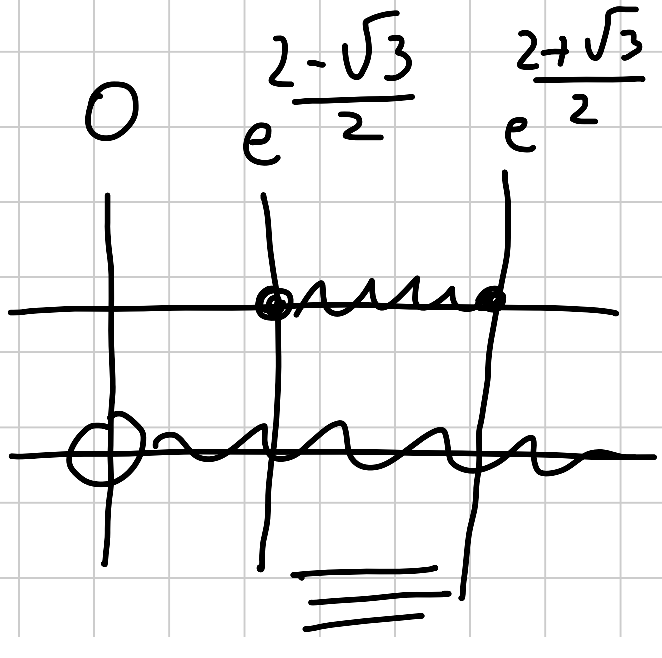
$$\frac{2-\sqrt{3}}{2} \leq \ln x \leq \frac{2+\sqrt{3}}{2}$$

$$\begin{cases} \ln x \ge \dfrac{2-\sqrt{3}}{2} \\ \ln x \le \dfrac{2+\sqrt{3}}{2} \end{cases} \qquad \begin{cases} \ln x \ge \ln\left(e^{\frac{2-\sqrt{3}}{2}}\right) \\ \ln x \le \ln\left(e^{\frac{2+\sqrt{3}}{2}}\right) \end{cases} \qquad \begin{cases} x \ge e^{\frac{2-\sqrt{3}}{2}} \\ x \le e^{\frac{2+\sqrt{3}}{2}} \end{cases}$$



$$CE \begin{cases} e^{\frac{2-\sqrt{3}}{2}} \le x \le e^{\frac{2+\sqrt{3}}{2}} \\ x > 0 \end{cases}$$

$$e^{\frac{2-\sqrt{3}}{2}} \le x \le e^{\frac{2+\sqrt{3}}{2}}$$



SCRIVI DOMINIO

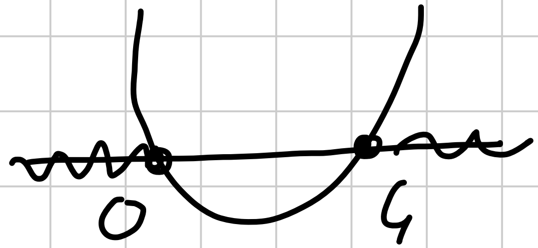
$$f(x) = \sqrt{x^2 - 4x}$$

$$x^2 - 4x \geq 0$$

$$x(x-4) = 0$$

$$x = 0$$

$$x = 4$$



$$x \leq 0 \quad \vee \quad x \geq 4$$

$$f(x) = \sqrt{3^{2x} - 4 \cdot 3^x + 3}$$

$$3^{2x} - 4 \cdot 3^x + 3 \geq 0$$

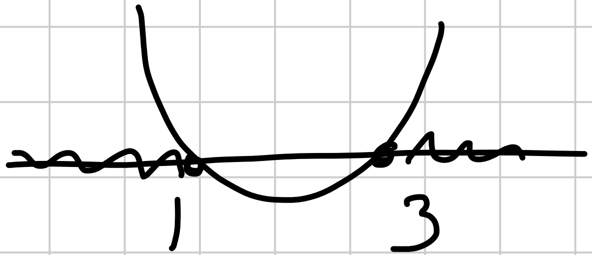
$$t = 3^x$$

$$t^2 - 4t + 3 \geq 0$$

$$t^2 - 4t + 3 = 0$$

$$(t-3)(t-1) = 0$$

$$t = 1 \quad \vee \quad t = 3$$



$$t \leq 1 \quad \vee \quad t \geq 3$$

$$3^x \leq 1 \quad \vee \quad 3^x \geq 3$$

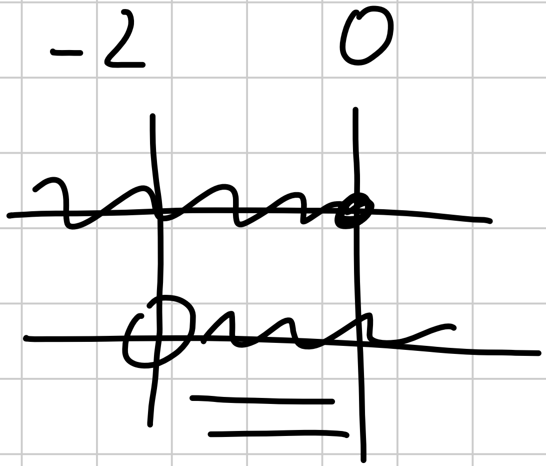
$$3^x \leq 3^0 \quad \vee \quad 3^x \geq 3^1$$

$$x \leq 0 \quad \vee \quad x \geq 1$$

$$f(x)=\sqrt{-x}+(2+x)^{-\frac{1}{2}}=\sqrt{-x}+\frac{1}{(2+x)^{\frac{1}{2}}}=$$

$$=\sqrt{-x}+\frac{1}{\sqrt{2+x}}$$

$$\begin{cases}-x\geq 0\\ 2+x>0\end{cases}\qquad\begin{cases}x\leq 0\\ x>-2\end{cases}$$



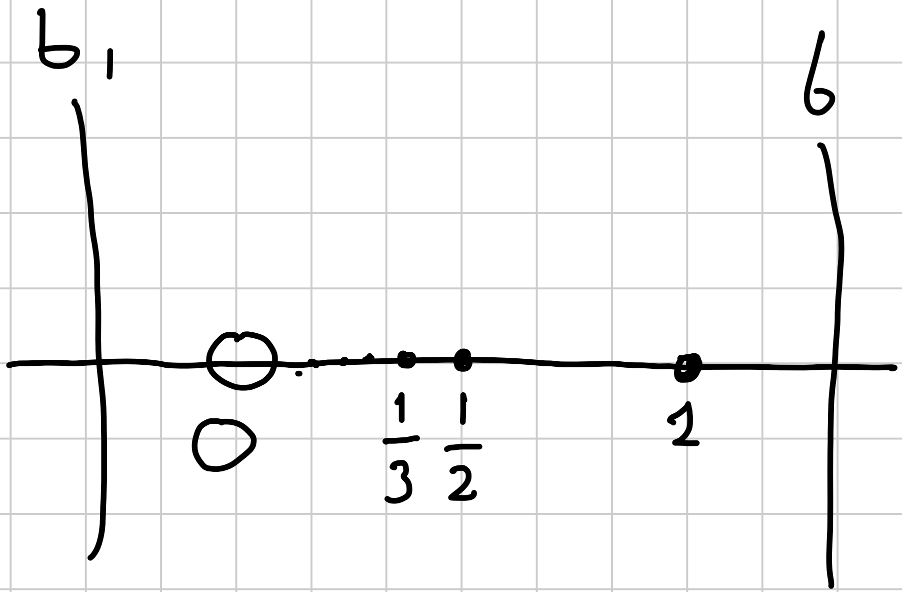
$$-2<x\leq 0$$

DETERMINA SUP / INF / MAX / MIN

$A \subseteq \mathbb{R}$

$\exists\ b \in \mathbb{R}$ t.c. $b \geq x \quad \forall x \in A$

(b è un maggiorante di A)



$$A = \left\{ \frac{1}{n},\ n \in \mathbb{N} \right\}$$

$$A = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \right\}$$

$$\frac{1}{n} \xrightarrow{n \to +\infty} 0$$

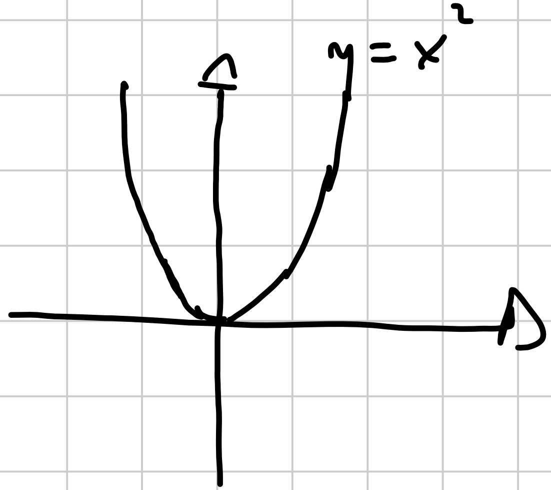
~~sup A~~ 2 è un maggiorante -1 è un minorante

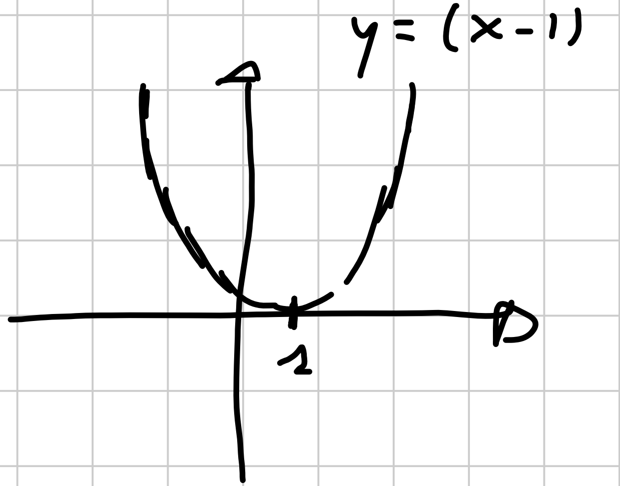
$\sup A = 1$ $\inf A = 0$

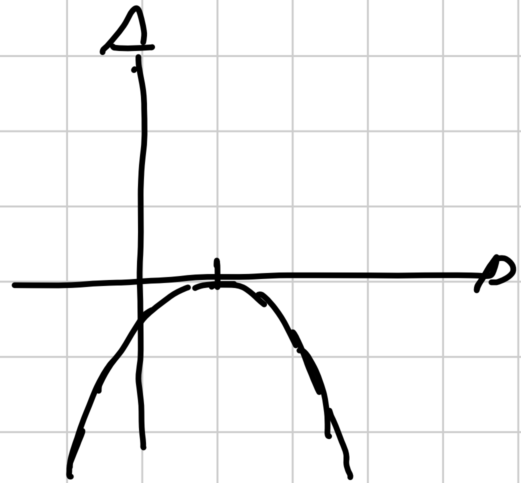
$\max A = 1$ $\min A = \nexists$

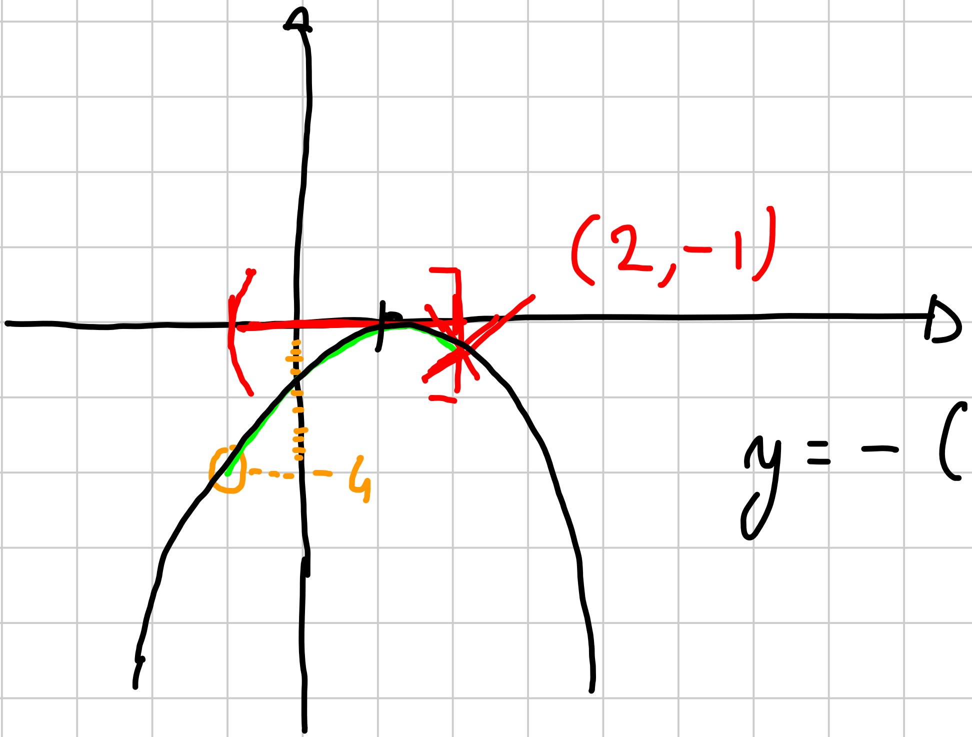
$f :]-1, 2] \to \mathbb{R}$ con $f(x) = -(x-1)^2$

$? = \sup / \inf / \max / \min$









$$y = -(x-1)^2$$

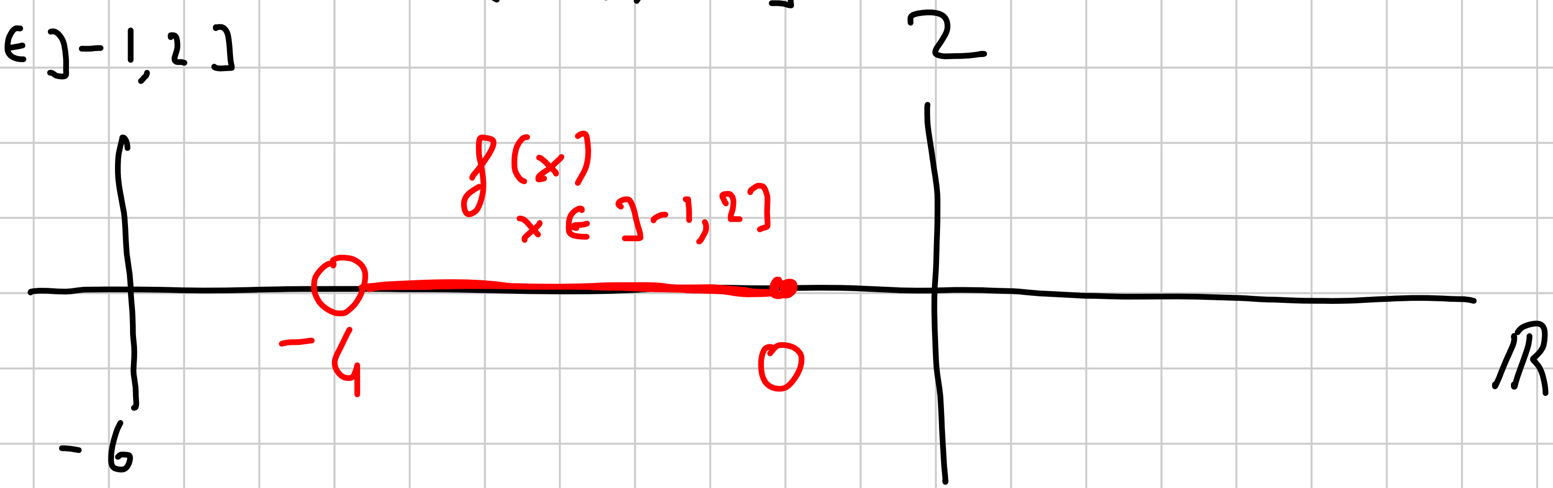
$$f :]-1, 2] \to \mathbb{R}$$

$$\cancel{f(x)} \text{ con}$$

$$f(-1) = -(-1-1)^2 = -4$$

$$f(2) = -(2-1)^2 = -1$$

$$\underset{x\in]-1,2]}{f(x)} = (-4, 0]$$



2 é maggiorante

-6 é un minorante

$$\sup_{x\in]-1,2]} f(x) = 0$$

$$\inf f(x) = -4$$

$$\max f(x) = 0$$

$$\min f(x) = \nexists$$

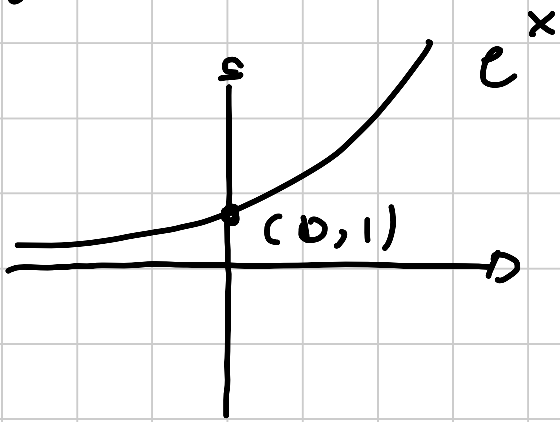
Determinare sup, inf, max, min di

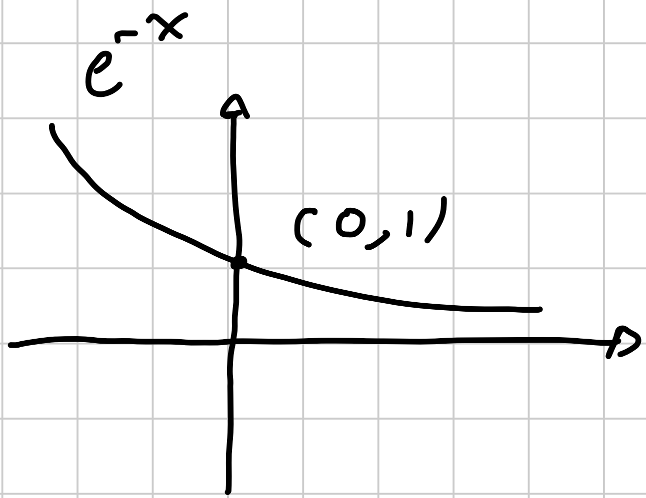
$$f(x) = e^{-x} - 1 \qquad x \in \mathbb{R}$$

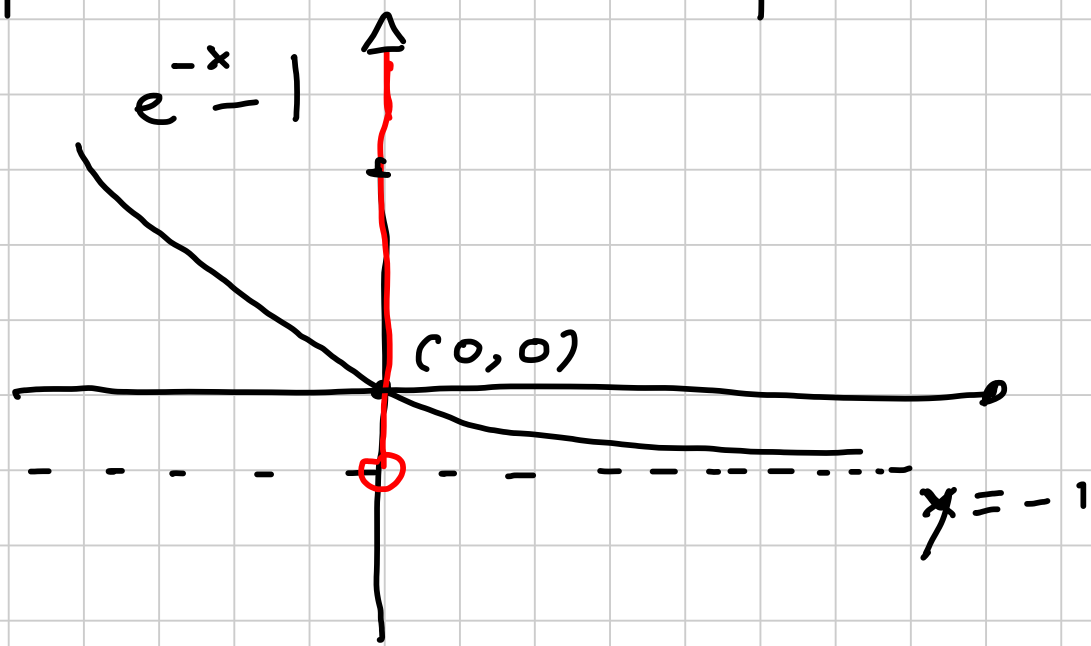
$f(x)$

- $f(-x) \rightarrow$ simmetrica rispetto asse y
- $-f(x) \rightarrow$ simmetrica asse x
- $f(x+k) \rightarrow$ trasla dx o sx
- $f(x)+k \rightarrow$ trasla alto o basso

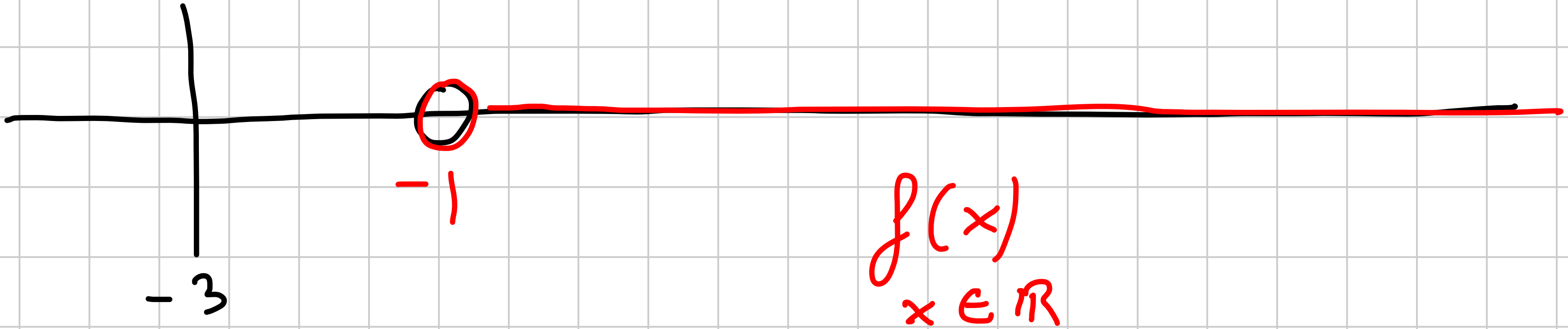
$f(x) = e^{-x} - 1$







$f(x) \in \left]-1, +\infty\right[$



$f(x)_{x \in \mathbb{R}}$

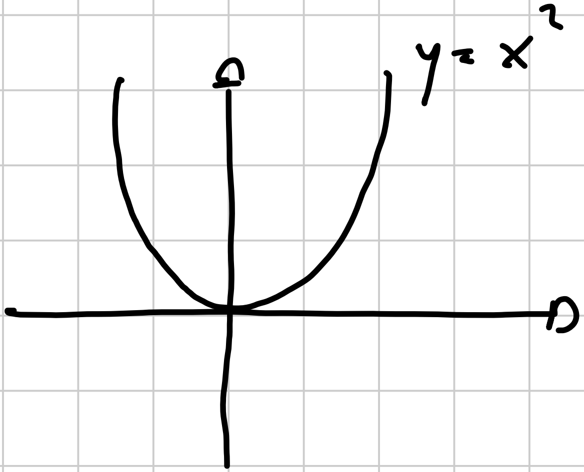
-3 è un minorante

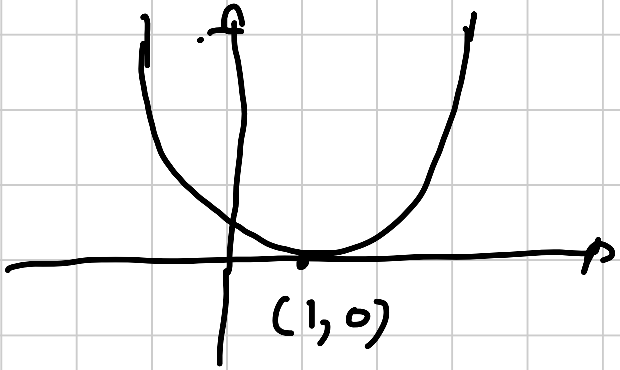
$\inf f(x) = -1$

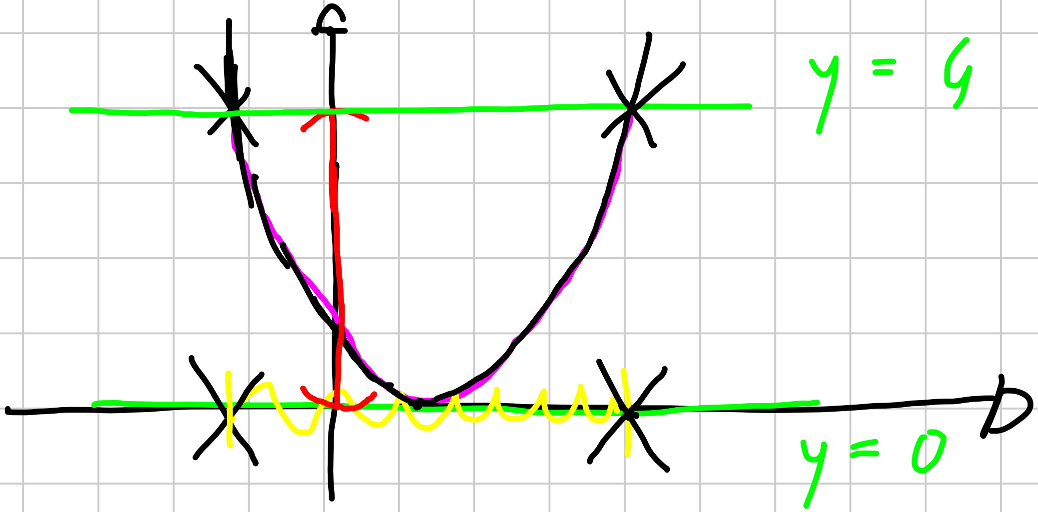
$\min f(x) = \nexists$

~~) Qet~~

•) Determina le controimmagine dell'intervallo $(0,4)$ attraverso la funzione $f(x) = (x-1)^2$



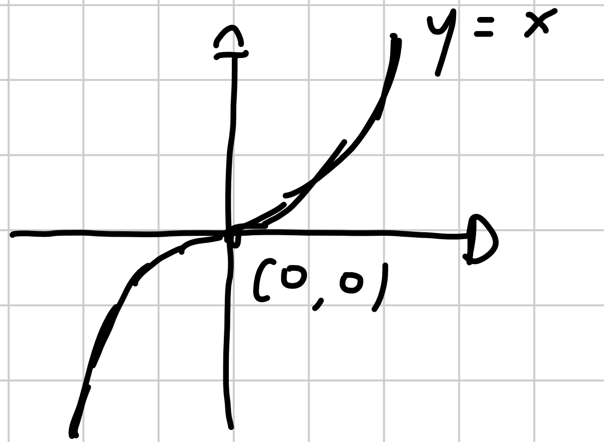


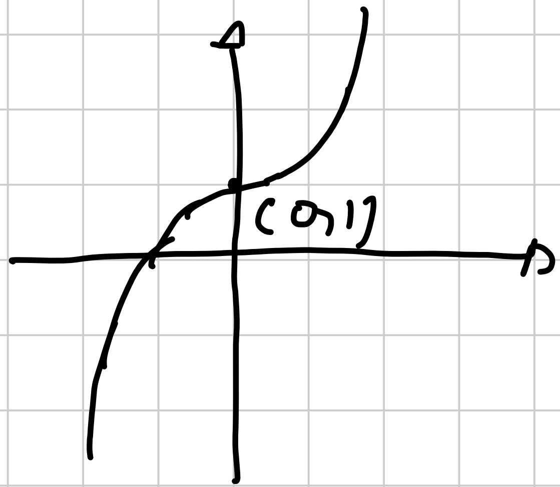


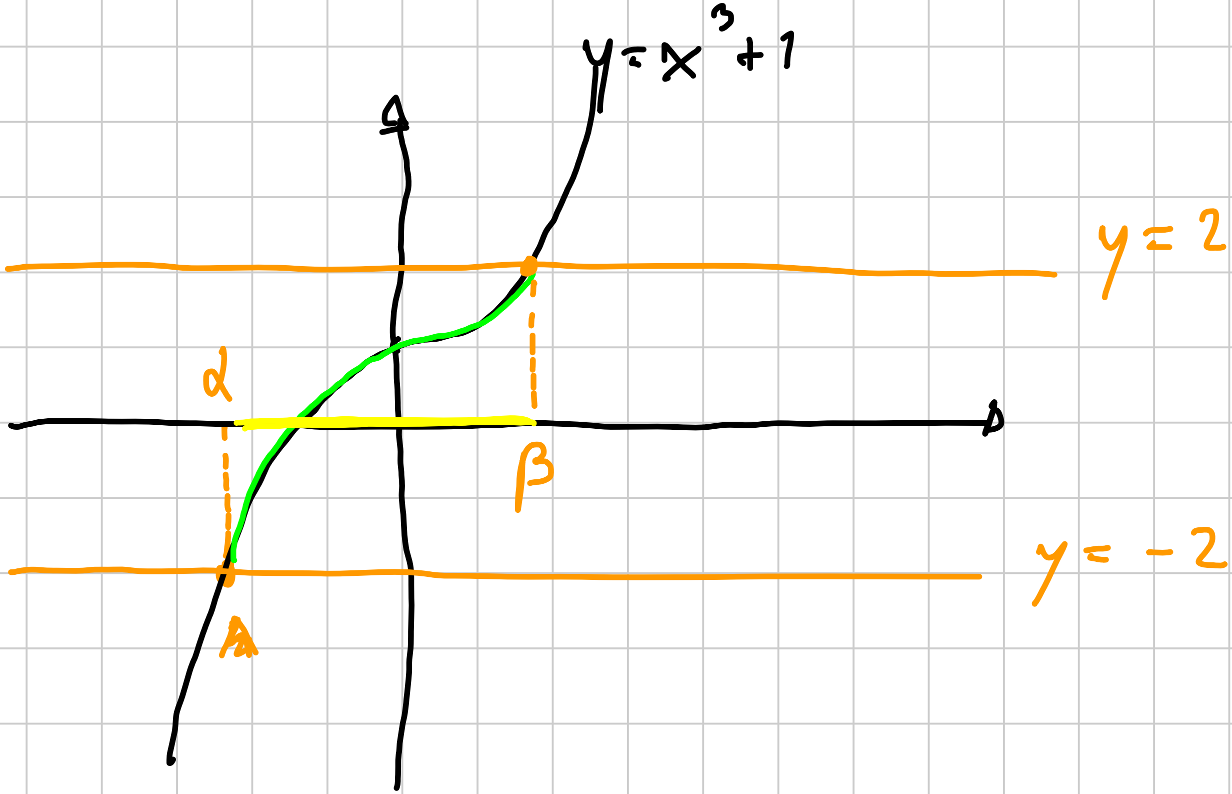
$$\begin{cases} f(x) > 0 \\ f(x) < 1 \end{cases} \qquad \begin{cases} (x-1)^2 > 0 \\ (x-1)^2 < 1 \end{cases}$$

$$\cancel{f(x) = 1}$$

ES: Determinare le controimmagine dell' intervallo $(-2,2)$ di $f(x) = x^3 + 1$







$\alpha < x < \beta$

Trovo α $\quad f(x) = -2$

$x^3 + 1 = -2$

$$x^3 = -3$$

$$x = \sqrt[3]{-3}$$

Trovo β

$$f(x) = 2$$

$$x^3 + 1 = 2$$

$$x^3 = 1$$

$$x = 1$$

$$\begin{cases} f(x) > -2 \\ f(x) < 2 \end{cases}$$