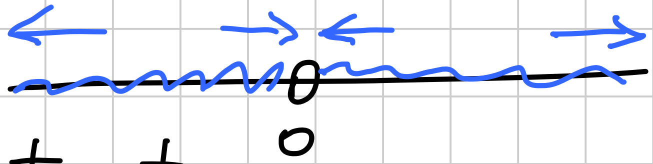
$$f(x) = \frac{1}{x^2} + e^{-x}$$

C.E. $x^2 \neq 0$; $x \neq 0$

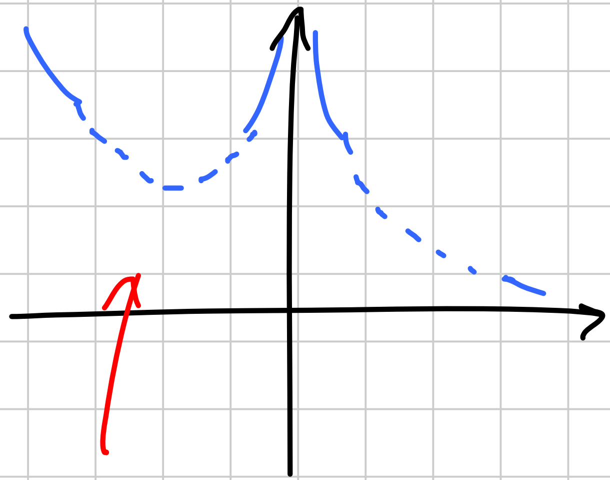


$$\lim_{x \to +\infty} f(x) = \frac{1}{+\infty} + e^{-\infty} = 0^+ + 0^+ = 0^+$$

$$\lim_{x \to -\infty} f(x) = \frac{1}{(-\infty)^2} + e^{+\infty} = 0^+ + \infty = +\infty$$

$$\lim_{x \to 0^+} f(x) = \frac{1}{0^+} + e^{0} = +\infty + 1 = +\infty$$

$$\lim_{x \to 0^-} f(x) = \frac{1}{(0^-)^2} + e^{0} = \frac{1}{0^+} + 1 = +\infty$$



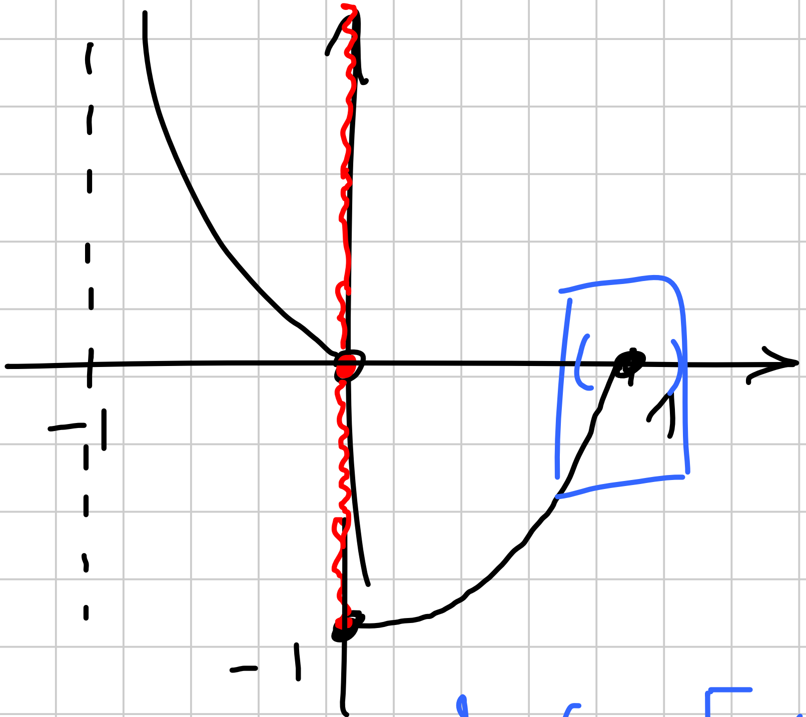
$$f'(x) = D x^{-2} + D e^{-x} = -2x^{-3} + e^{-x} \cdot (-1)$$
$$= -\frac{2}{x^3} - e^{-x}$$

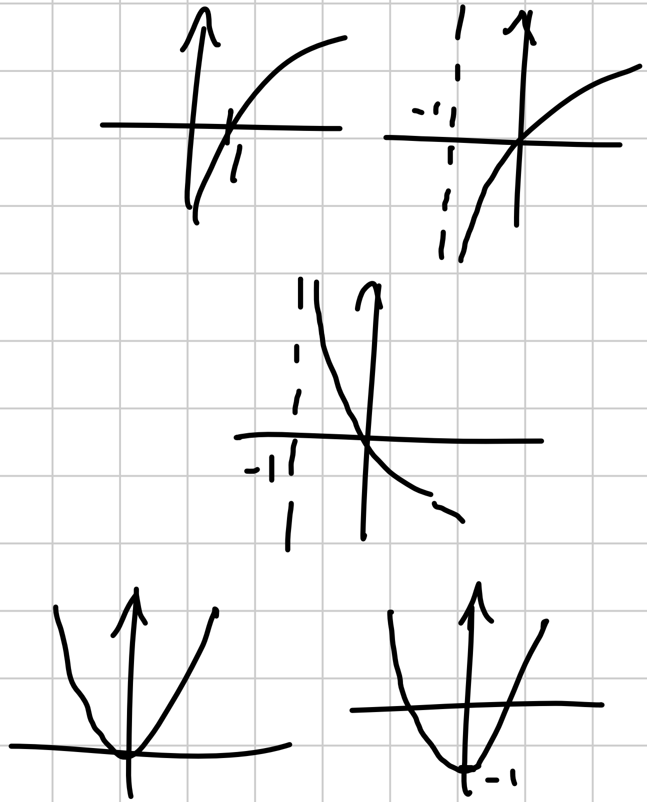
se $x > 0$ $f'(x) < 0$

$x < 0$

$$\underbrace{-\frac{2}{x^3}}_{+} \underbrace{- e^{-x}}_{-}$$

$$f(x)=\begin{cases} -\ln(x+1) & -1<x<0 \\ x^2-1 & 0\le x\le 1 \end{cases}$$





$\operatorname{Im} f = [-1, +\infty)$

pto di min (globale) : $x=0$ $\quad (f(0)=-1)$

$\nexists$ pto di max globale ; pto di max locale $x=1$

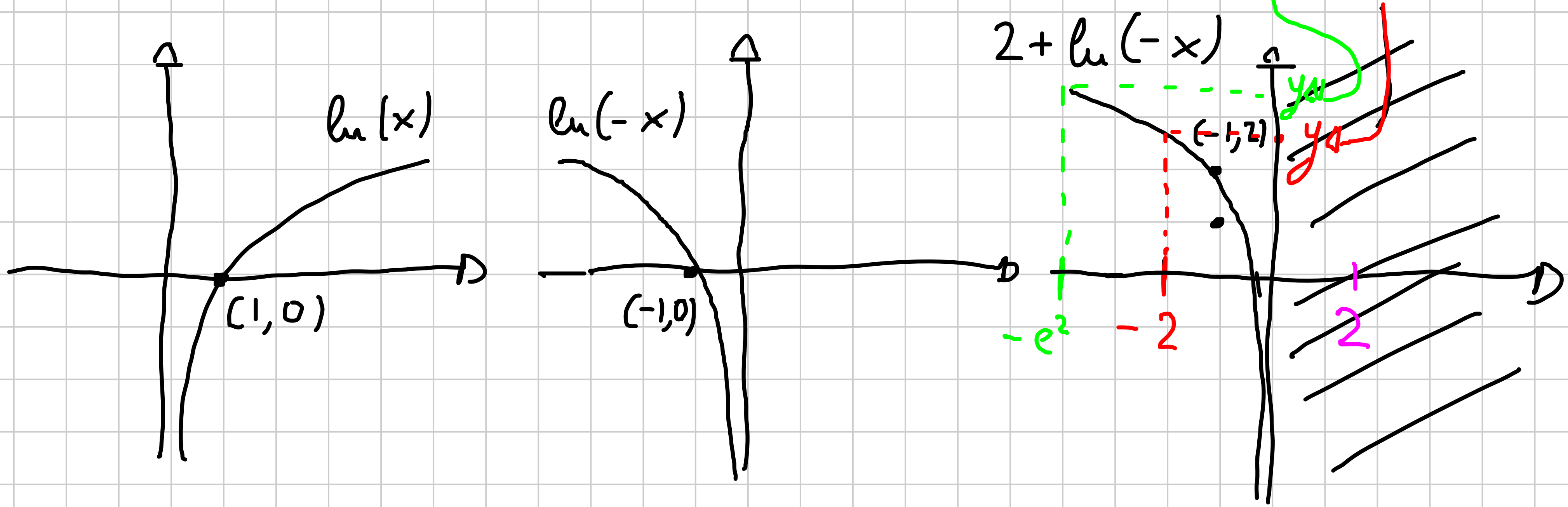
$\nexists$ pti di min locale (a parte $x=0$)

Immagine / controimmagine

CE $-x > 0$

$x < 0$

$$f(x) = 2 + \ln(-x)$$

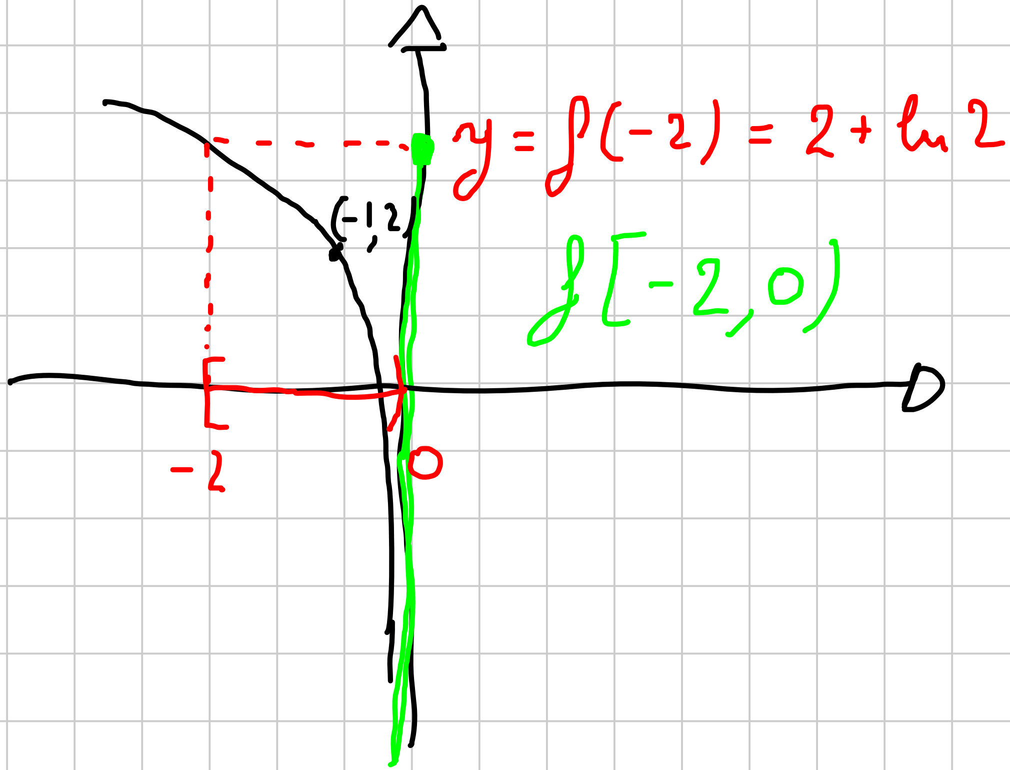


$$f(-2) = 2 + \ln(-(-2)) = 2 + \ln 2$$

$$f(-e^2) = 2 + \ln(-(-e^2)) = 2 + \ln e^2 = 2 + 2 = 4$$

$$f(2) = \nexists$$

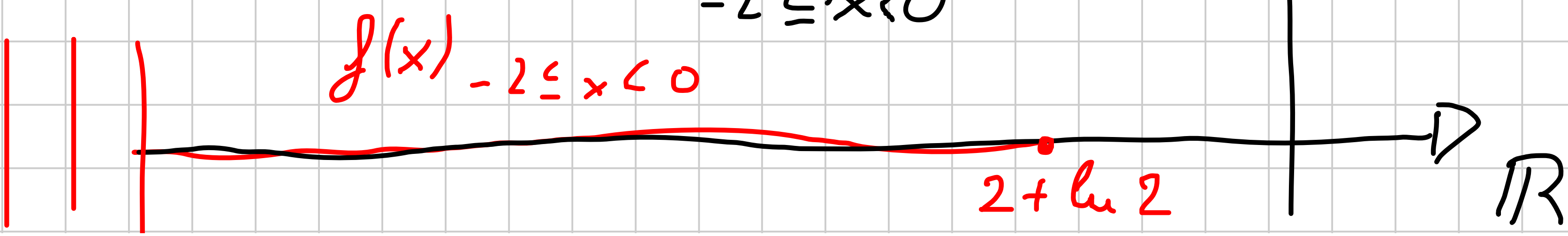
$f[-2,0)$



"$f(0) = -\infty$"

$$f[-2,0) = (-\infty, 2+\ln 2]$$

$$\sup/\inf/\max/\min \underset{-2\leq x<0}{f(x)}$$



$3 + \ln 2$ è un maggiorante di $\underset{-2 \le x < 0}{f(x)}$

$\sup f(x) = 2 + \ln 2$

$\max f(x) = 2 + \ln 2$

no inferiormente limitata

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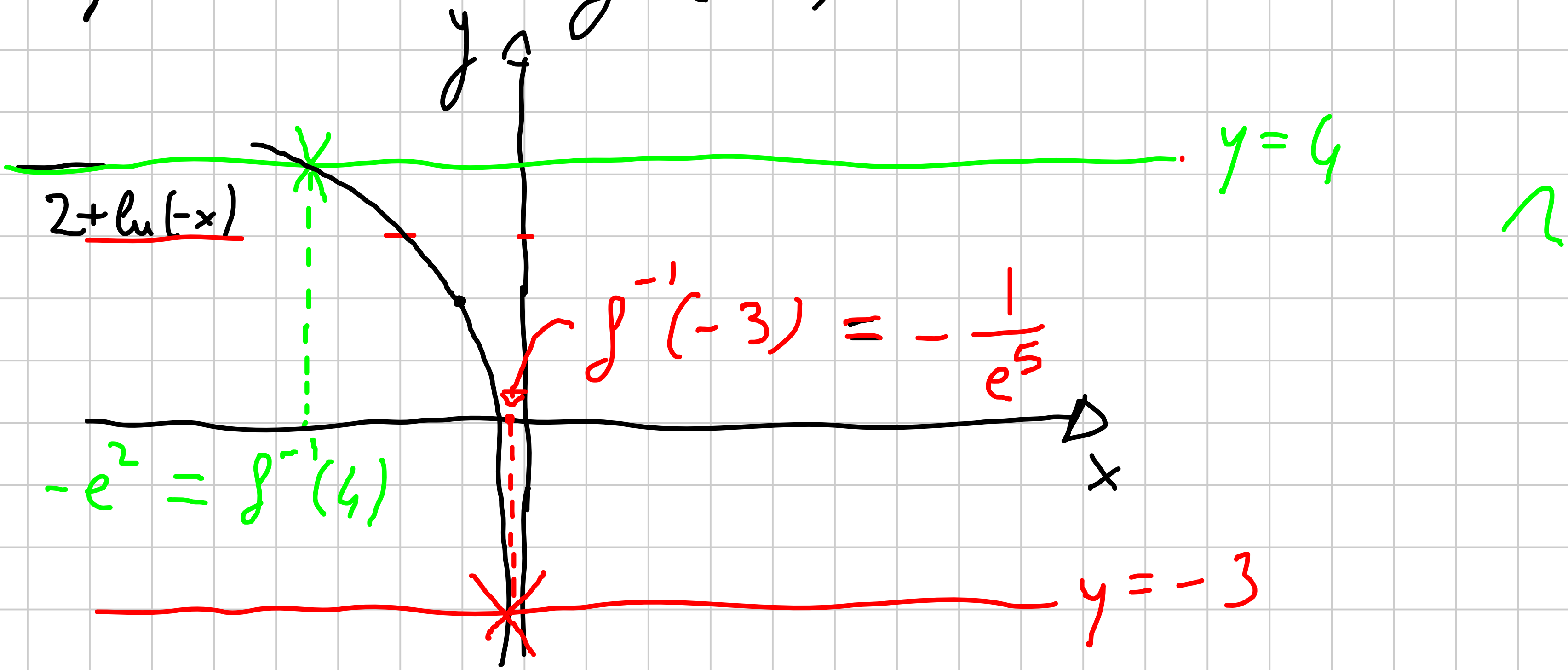
controimmagine -3 $f^{-1}(-3)$



$$f(x) = -3$$

$$2 + \ln(-x) = -3$$

$$\ln(-x) = -5$$

$$\ln(-x) = \ln(e^{-5})$$

$$-x = e^{-5}$$

$$x = -e^{-5}$$

$$x = -\frac{1}{e^5}$$

$$\boxed{\begin{aligned} -5 &= \ln z \\ z &= e^{-5} \\ -5 &= \ln e^{-5} \end{aligned}}$$

$$f^{-1}(4) = ?$$

$$f(x) = 4$$

$$2 + \ln(-x) = \cancel{2}\ 4$$

$$\ln(-x) = 2$$

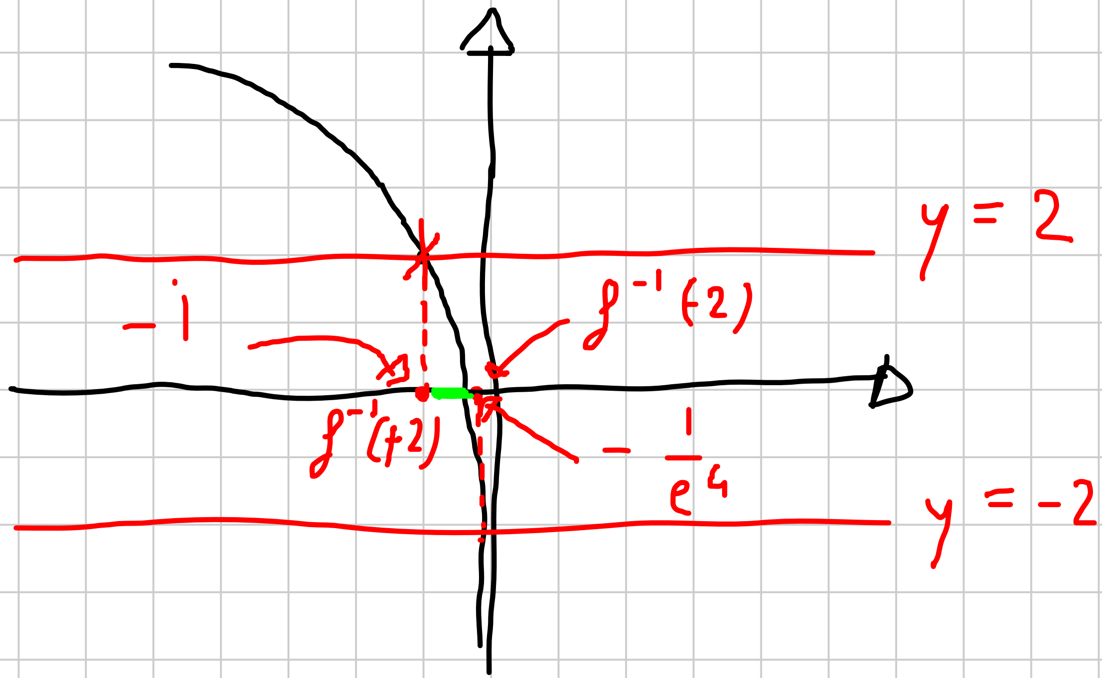
$$\ln(-x) = \ln e^2$$

$$-x = e^2$$

$$x = -e^2$$

$$f^{-1}(4) = -e^2$$

$$f^{-1}[-2, 2]$$



$$f^{-1}(-2) = \ ?$$

$$f(x) = -2$$

$$2 + \ln(-x) = -2$$

$$\ln(-x) = -4$$

$$\ln(-x) = \ln e^{-4}$$

$$-x = e^{-4}$$

$$x = -\frac{1}{e^4}$$

$$f^{-1}(2) = \ ?$$

$$f(x) = 2$$

$$2 + \ln(-x) = 2$$

$$\ln(-x) = 0$$

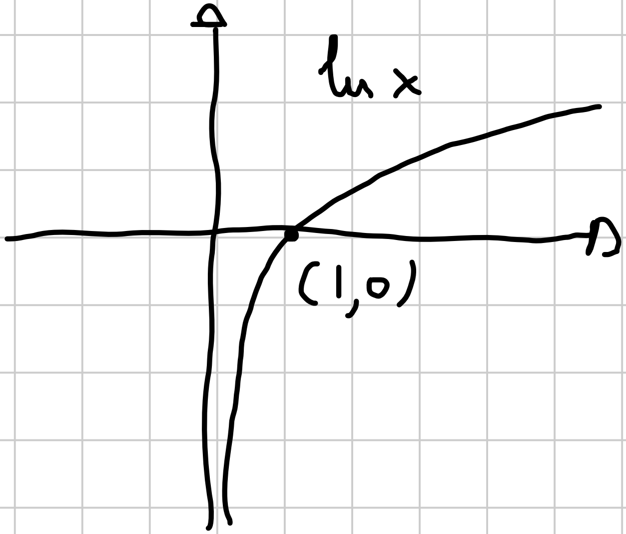
$$\ln(-x) = \ln e^0$$

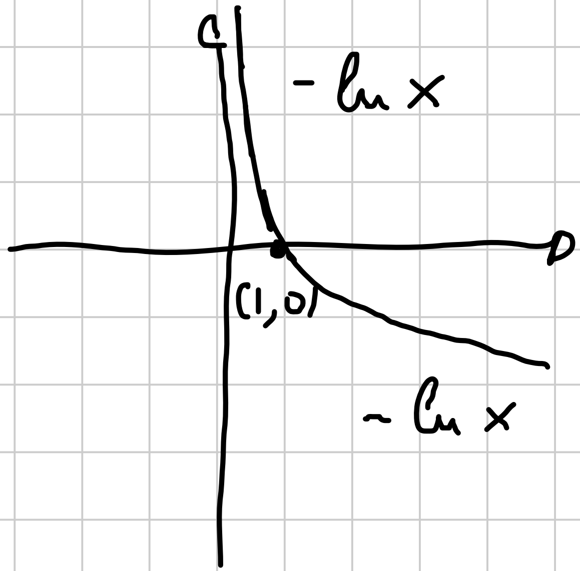
$$-x = e^0 = 1$$

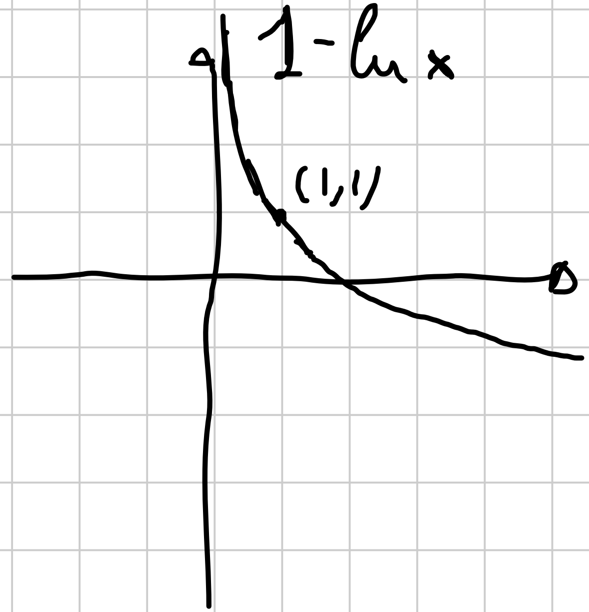
$$x = -1$$

$$f^{-1}[-2, 2] = \left[-1, -\frac{1}{e^4}\right]$$

$$f(x) = |1 - \ln x| \qquad CE: x > 0$$







$$f^{-1}(0) = ?$$

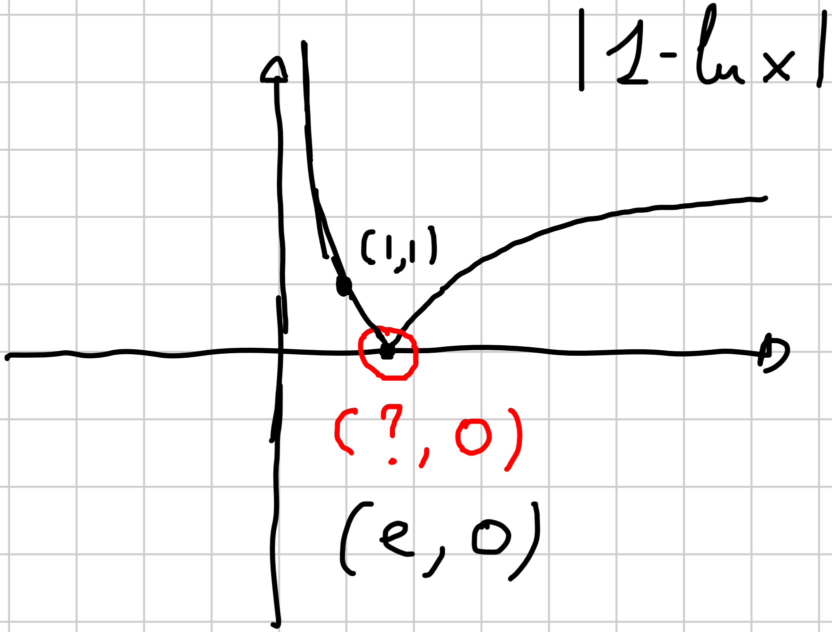
$$f(x) = 0$$

$$|1 - \ln x| = 0$$

$$1 - \ln x = 0$$

$$+\ln x = +1$$

$$+\ln x = \ln e^{1}$$



$x = e^{1} =$

$1 = \ln y$

$y = e^{1}$

$1 = \ln e^{1}$

$\cancel{f(1)}$ $f(x) = |1 - \ln x|$

$$f(x) = \begin{cases} 1 - \ln x & 1 - \ln x \geq 0 \\ -1 + \ln x & 1 - \ln x < 0 \end{cases}$$

$$|x| \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$f(x) = |1 - \ln x|$

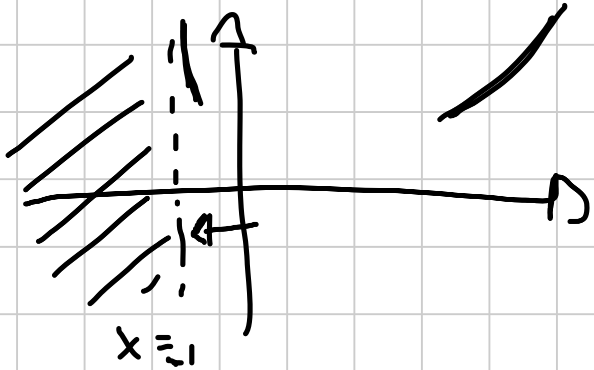
$f(1) = |1 - \ln 1| = |1 - 0| = 1$

$f(\cdot) = |-2| = 2$

$f(2) = |1 - \ln 2| = |\pm num| = + num$

LIMITI

$f(x) = x^2 - \ln(x+1)$ $D: x > -1$



$$\lim_{x \to -1^+} \underbrace{x^2}_{\downarrow\, 1} - \underbrace{\ln(\underbrace{x+1}_{\downarrow\, 0^+})}_{\to -\infty} \overset{1+\infty}{=} +\infty$$

$$\lim_{x \to +\infty} \underbrace{x^2}_{\downarrow\, +\infty} - \underbrace{\ln(\underbrace{x+1}_{\downarrow\, +\infty})}_{\downarrow\, +\infty} \overset{+\infty-\infty}{=} \lim_{x \to +\infty} x^2 = +\infty$$

$$f'(x) = 2x - \frac{1}{x+1} = \frac{2x(x+1) - 1}{x+1} = \frac{2x^2 + 2x - 1}{x+1}$$

CE $\boxed{x \neq -1}$

$$f'(x) \geq 0$$

$N \geq 0 \qquad 2x^2 + 2x - 1 \geq 0$

$$12 = 2^2 \cdot 3$$
$$\sqrt{12} = 2\sqrt{3}$$

$$2x^2 + 2x - 1 = 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{4+8}}{4} = \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} =$$

$$= \frac{2(-1 \pm \sqrt{3})}{2} = \frac{-1 \pm \sqrt{3}}{2}$$

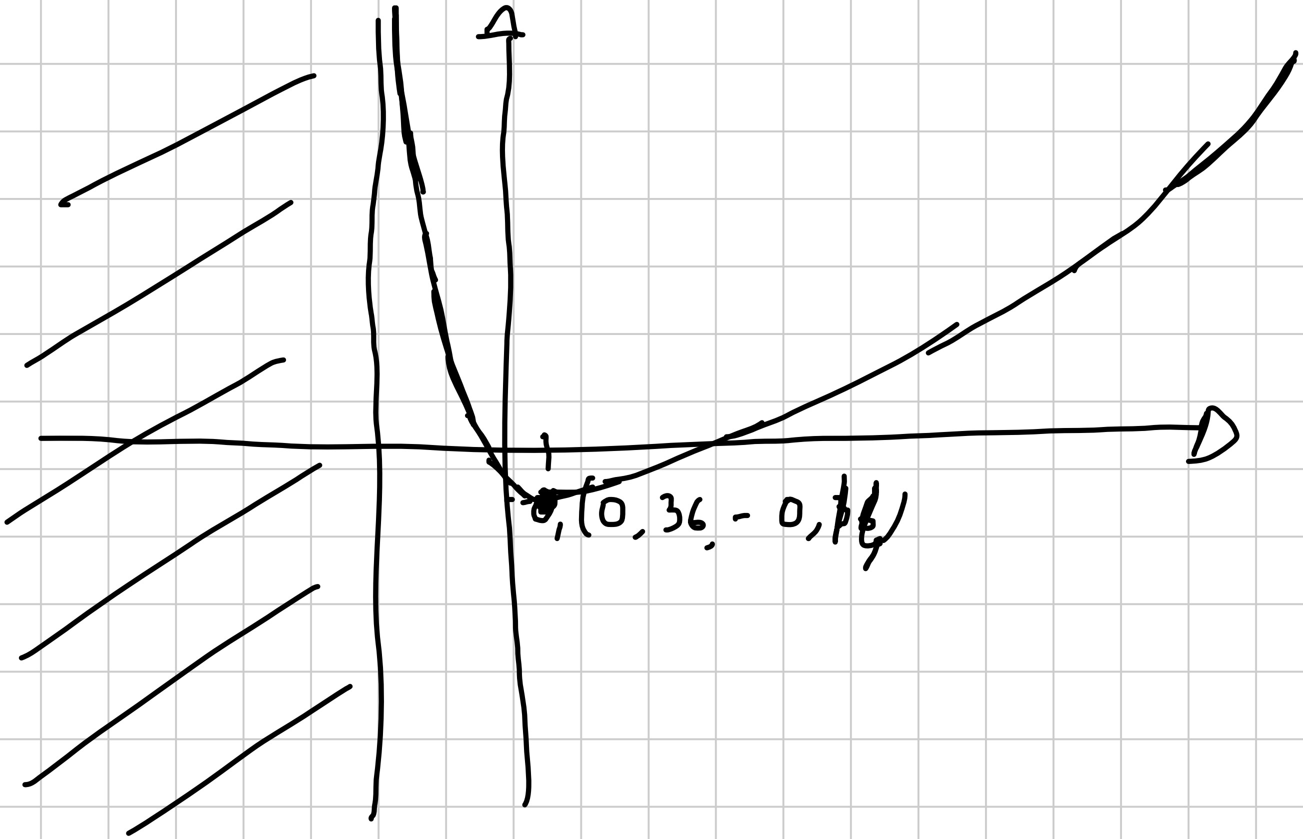
$$x = \frac{-1-\sqrt{3}}{2} \quad \vee \quad x = \frac{-1+\sqrt{3}}{2}$$

$N \geq 0 \qquad x \leq \frac{-1-\sqrt{3}}{2} \quad \vee \quad x > \frac{-1+\sqrt{3}}{2}$

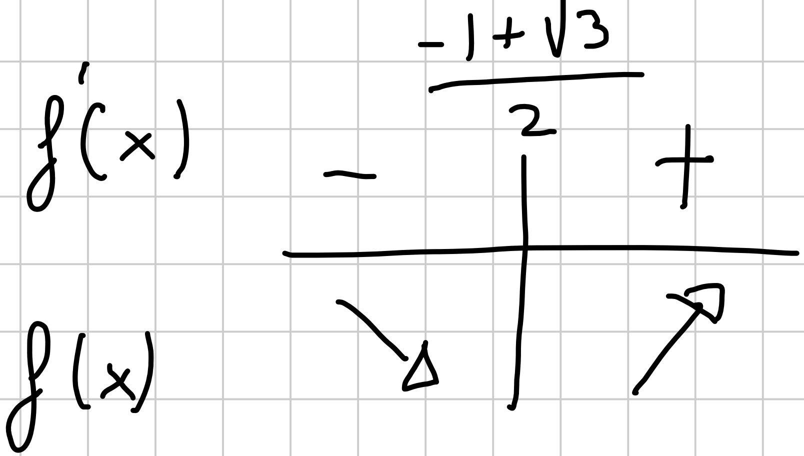
$D > 0 \qquad x+1 > 0 \rightarrow x > -1$

		$\frac{-1-\sqrt{3}}{2}$		-1		$\frac{-1+\sqrt{3}}{2}$	
N	$+$		$-$		$-$		$+$
D	$-$		$-$		$+$		$+$
$\frac{N}{D}$					$-$		$+$

$f'(x) \geq 0 \qquad x > \frac{-1+\sqrt{3}}{2} = 0{,}36.$



$x = -1$



$$x_{min} = \frac{-1+\sqrt{3}}{2}$$

$$y_{min} = f\left(\frac{-1+\sqrt{3}}{2}\right) = \left(\frac{-1+\sqrt{3}}{2}\right)^2 - \ln\left(\frac{-1+\sqrt{3}}{2} + 1\right)$$

$$= \frac{(\sqrt{3}-1)^2}{4} - \ln\left(\frac{-1+\sqrt{3}+2}{2}\right)$$

$$= \frac{(\sqrt{3}-1)^2}{2} - \ln\left(\frac{\sqrt{3}+1}{2}\right) = -0{,}\cancel{3}4$$

$$f'(x) = \frac{2x^2+2x-1}{x+1} \qquad \frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g(x)^2}$$

$$f''(x) = \frac{(4x+2)(x+1) - (2x^2+2x-1)\cdot 1}{(x+1)^2}$$

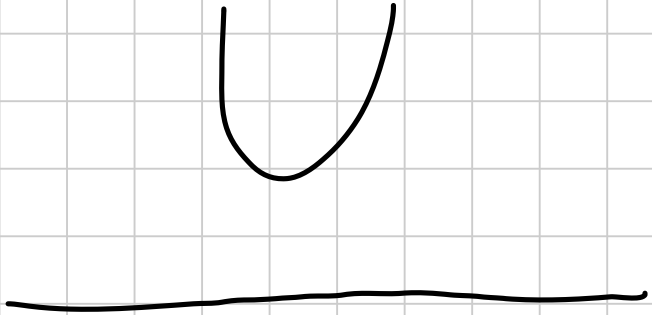
$$= \frac{4x^2+\cancel{2x}+4x+2-2x^2-\cancel{2x}+1}{(x+1)^2}$$

$$= \frac{2x^2+4x+3}{(x+1)^2}$$

$$f''(x) \geq 0$$

$$N \geq 0 \qquad 2x^2+4x+3 \geq 0$$

$$x_{1,2} = -4 \pm \sqrt{16-24} \qquad \Delta < 0$$



$$2x^2+4x+3 = 0$$