

$$\lim_{x \rightarrow 0^+} \left( \frac{\sqrt{x}}{e^x} + \frac{x+1}{\sqrt{x}} \right) = +\infty$$

Handwritten annotations in red:

- $\sqrt{x} \rightarrow 0^+$
- $e^x \rightarrow 1^+$
- $\frac{\sqrt{x}}{e^x} \rightarrow 0$
- $\frac{x+1}{\sqrt{x}} \rightarrow +\infty$

$$f(x) = (3x+4)^2 e^{3x+4}$$

$$g(x) = (3x+4)^2$$

$$h(x) = e^{3x+4}$$

$$f(x) = g(x) \cdot h(x)$$

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

$$g(x) = (3x + 4)^2$$

Diagram illustrating the substitution method:

$x$  is mapped to  $3x + 4 = t$  (labeled  $f$ ), and  $t$  is mapped to  $t^2$  (labeled  $g$ ).

$$g'(x) = \frac{d}{dx}(3x + 4) \cdot \frac{d}{dt}(t^2)$$

$$= 3 \cdot 2t = 6t = 6(3x + 4)$$

Diagram illustrating the composition method:

$x$  is mapped to  $3x + 4 = t$  (labeled  $f(x)$ ), and  $t$  is mapped to  $t^2 = (3x + 4)^2$  (labeled  $g(t)$ ).

$$(3x + 4)^2 = g(f(x))$$

$$g(f(x))' = \frac{df}{dx} \cdot \frac{dg(f(x))}{df(x)}$$

| = 3

$$f(x) = 3x + 4$$

$$(3x + 4)^3 = z^3$$

$$g'(x) = 3 \cdot 2(3x + 4)$$

$$h'(x) = 3e^{3x+4}$$

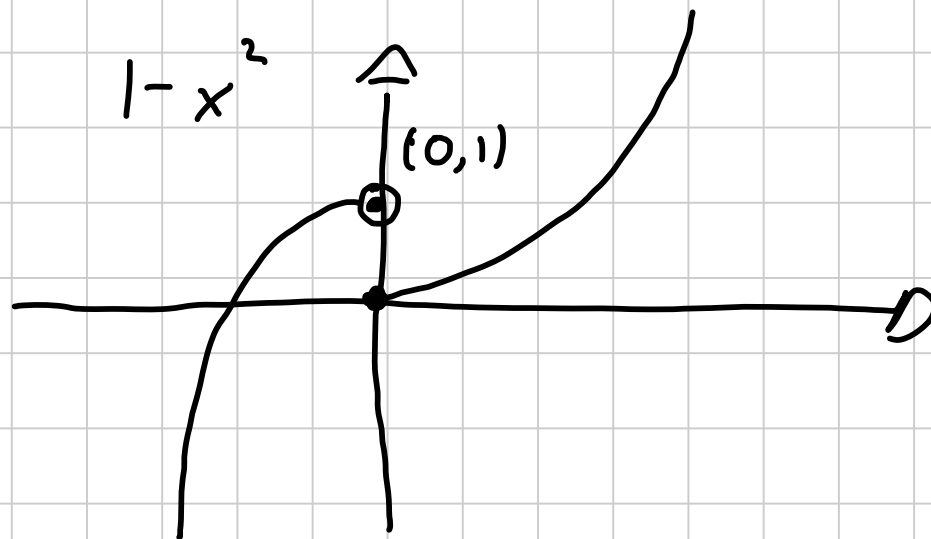
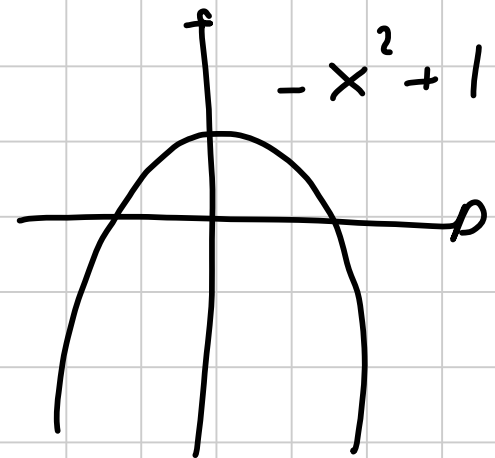
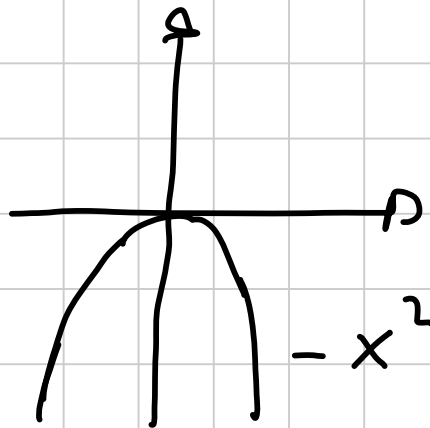
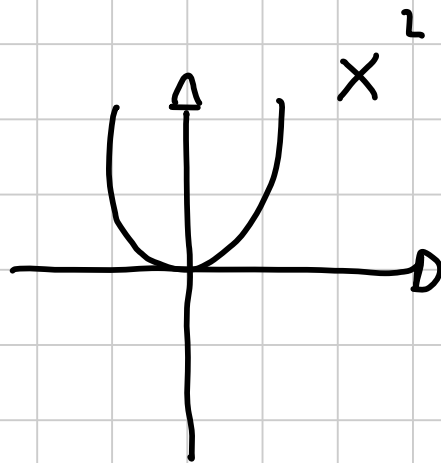
$$h(x) = e^{3x+4}$$

$$x \xrightarrow{3x+4=t} \frac{d(3x+4)}{dx} \cdot \frac{de^t}{dt}$$

$$f'(x) = 6(3x+4)e^{3x+4} + (3x+4)^2 \cdot 3e^{3x+4}$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = \begin{cases} 1-x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

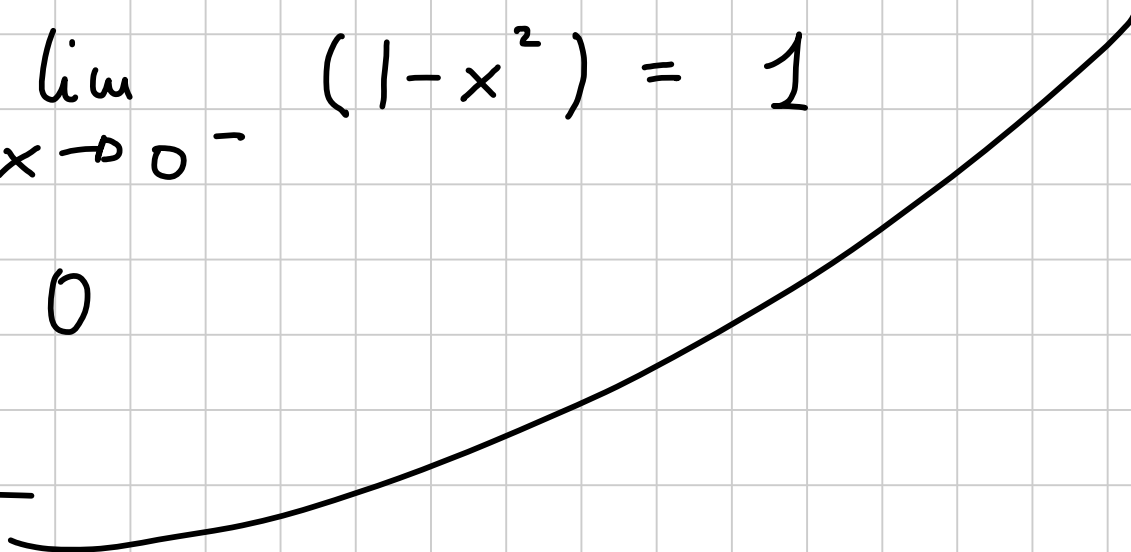


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$$\lim_{x \rightarrow 0^-} f(x) \quad / \quad \lim_{x \rightarrow 0^+} f(x) \quad / \quad \underline{f(0) = 0}$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (1 - x^2) = 1$$

$$\lim_{x \rightarrow 0^+} x^2 = 0$$



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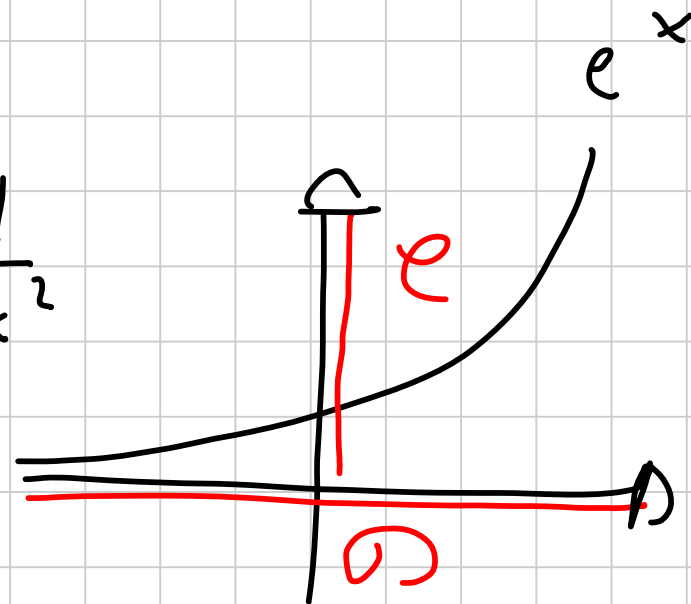
# ESERCIZIO 3

$$f(x) = e^{-x}$$

$$g(x) = \frac{1}{x^2}$$

$$f(g(x))$$

$$g(f(x))$$



$$f(g(x))$$

$$x \xrightarrow{g(x)} \frac{1}{x^2} = t$$

$$f(t)$$

$$e^{-t} = e^{-\frac{1}{x^2}}$$

0) :  $x \neq 0$

?  $e : y > 0$   
?

$$\begin{array}{c}
 g(f(x)) \\
 \uparrow f(x) \\
 x \rightarrow e^{-x} = z
 \end{array}
 \quad
 \begin{array}{c}
 g(z) \\
 \uparrow \\
 \frac{1}{z^2} = \frac{1}{(e^{-x})^2} = \frac{1}{e^{-2x}}
 \end{array}$$

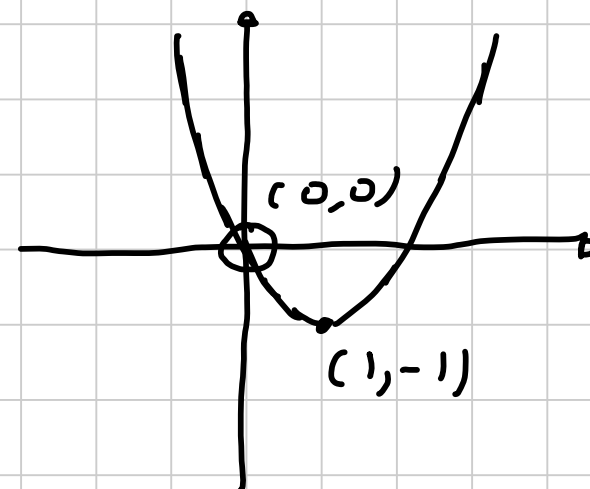
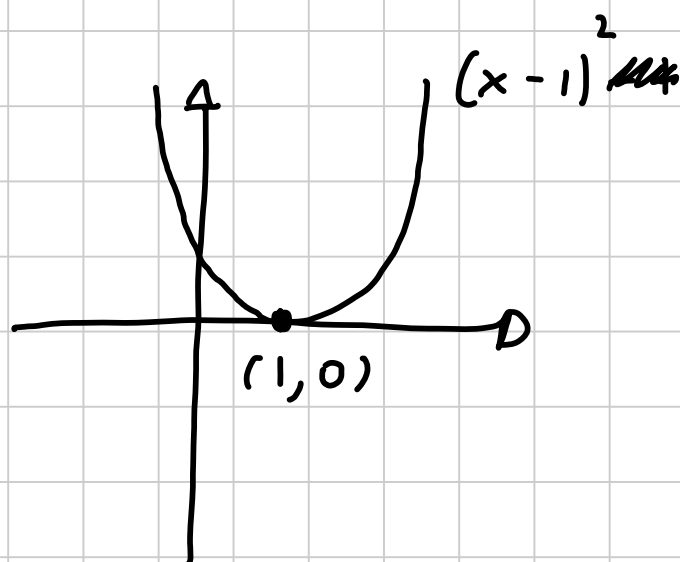
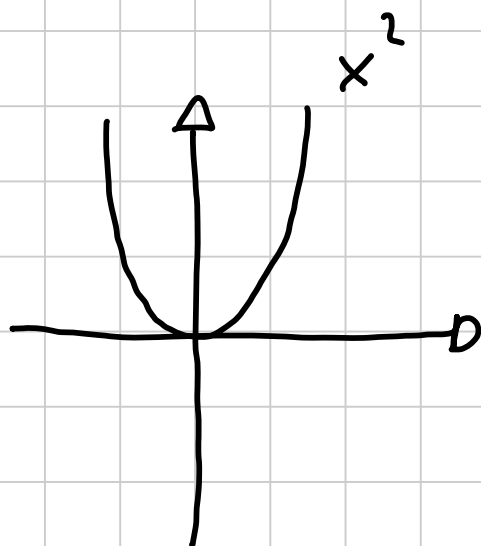
$$\begin{array}{l}
 \lim_{x \rightarrow +\infty} \frac{f(x)}{g(x)} = 0 \\
 \lim_{x \rightarrow +\infty} \frac{e^{-x}}{\frac{1}{x^2}} = \lim_{x \rightarrow +\infty} e^{-x} \cdot x^2 = \lim_{x \rightarrow +\infty} \frac{x^2}{e^x}
 \end{array}$$

Annotations: Red arrows indicate limits of components:  $e^{-x} \rightarrow 0$ ,  $\frac{1}{x^2} \rightarrow 0$ ,  $e^{-x} \rightarrow 0$ ,  $x^2 \rightarrow +\infty$ ,  $\frac{x^2}{e^x} \rightarrow \frac{\infty}{\infty}$ .

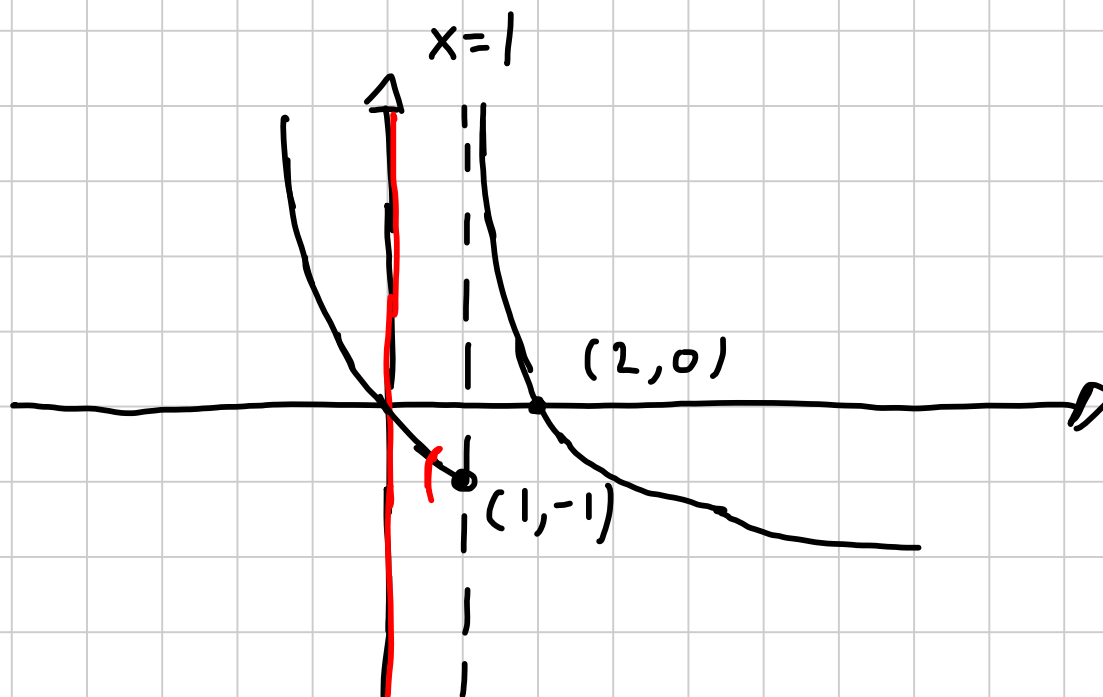
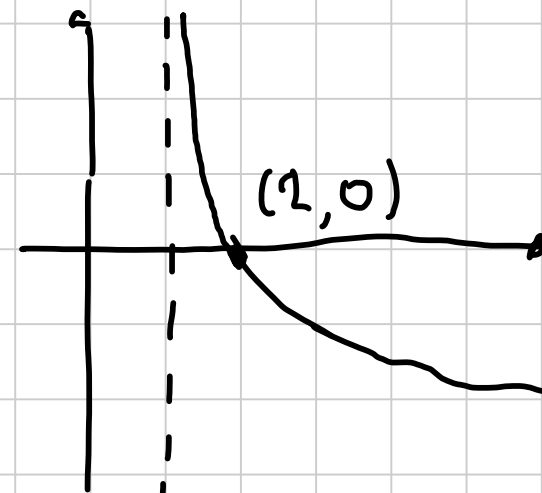
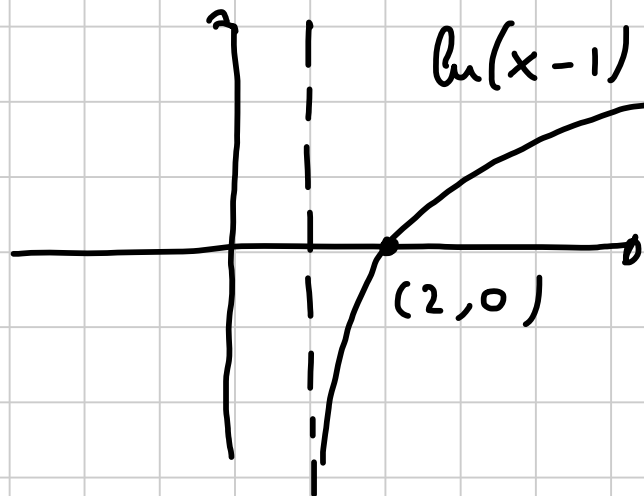
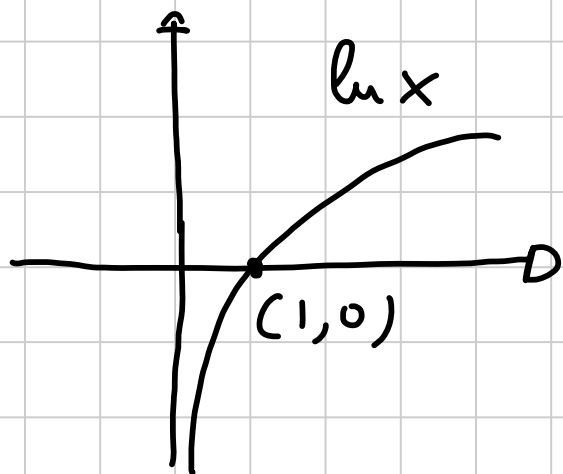
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## ESERCIZIO 1

$$f(x) = \begin{cases} (x-1)^2 - 1 & x \leq 1 \\ -\ln(x-1) & x > 1 \end{cases}$$







derivabile  $\Rightarrow$  continua

continuous  $\nRightarrow$  derivabile

non continua  $\Rightarrow$  non derivabile

17/09/2013

# Esercizio 1

$$f(x) = \ln\left(\frac{x-1}{x}\right) - x$$

$$b) : \frac{x-1}{x} > 0 \quad \begin{array}{l} N > 0 \\ D > 0 \end{array} \quad \begin{array}{l} x > 1 \\ x > 0 \end{array}$$

| 0 | 1 |
|---|---|
| - | + |
| - | + |
| + | + |

$x < 0$        $x > 1$

+

$$\left( \frac{1+\sqrt{5}}{2}, f\left(\frac{1+\sqrt{5}}{2}\right) \right)$$

$$\frac{1-\sqrt{5}}{2} A$$

$$B \frac{1+\sqrt{5}}{2}$$

$$A \left( \frac{1-\sqrt{5}}{2}, f\left(\frac{1-\sqrt{5}}{2}\right) \right)$$

$$x=1$$

—

$$\lim_{x \rightarrow -\infty} \ln \left( \frac{x-1}{x} \right) - x = \lim_{x \rightarrow -\infty} \ln \left( \frac{x}{x} \right) - x =$$

$$= \lim_{x \rightarrow -\infty} \underbrace{\ln 1}_{0} - \underbrace{x}_{+\infty} = +\infty$$

$$\lim_{x \rightarrow 0^-} \ln \left( \frac{\overbrace{x-1}^{-1}}{\underbrace{x}_{0^-}} \right) - \underbrace{x}_{0^-} = +\infty \quad -0,1-1 = -1,1$$

$\underbrace{\quad}_{+\infty}$   
 $\underbrace{\quad}_{+\infty}$

$$\lim_{x \rightarrow 1^+} \ln \left( \frac{x-1}{x} \right) - x = -\infty \quad 1, 1-1 = 0, 1$$

$\underbrace{\quad}_{1^+}$   
 $\underbrace{\quad}_{0^+}$   
 $-\infty$

$$\lim_{x \rightarrow +\infty} \ln \left( \frac{x-1}{x} \right) - x = \lim_{x \rightarrow +\infty} -x = -\infty$$

$$f(x) = \ln \left( \frac{x-1}{x} \right) - x$$

Intersezioni asse x

$$f(x) = 0 \rightarrow \ln \left( \frac{x-1}{x} \right) - x = 0$$

## DERIVATA PRIMA

$$f(x) = \ln\left(\frac{x-1}{x}\right) - x$$

~~2/2~~

$$D f(x) = D\left[\ln\left(\frac{x-1}{x}\right)\right] - D[x]$$

$$D\left[\ln\left(\frac{x-1}{x}\right)\right] = D\left[\frac{x-1}{x}\right] \cdot D[\ln t]$$

$x \xrightarrow{g(x)} \underbrace{\frac{x-1}{x}}_{=t}$

$\xrightarrow{h(t)} \underbrace{\ln t}$

$$D \left[ \frac{x-1}{x} \right] = \frac{1x - (x-1)}{x^2} = + \frac{1}{x^2}$$

$$D [\ln t] = \frac{1}{t} = \frac{1}{\frac{x-1}{x}} = \frac{x}{x-1}$$

$$D \left[ \ln \left( \frac{x-1}{x} \right) \right] = + \frac{1}{x^2} \cdot \frac{x}{x-1} = + \frac{1}{x(x-1)}$$

$$D \left[ \ln \left( \frac{x-1}{x} \right) - x \right] = + \frac{1}{x(x-1)} - 1$$

$$f'(x) > 0$$

$$\frac{1}{x(x-1)} - 1 > 0$$

$$\frac{1 - x(x-1)}{x(x-1)} > 0$$

$$\frac{1 - x^2 + x}{x^2 - x} > 0 \rightarrow \frac{-x^2 + x + 1}{x^2 - x} > 0$$

$$N > 0 \quad \overline{-x^2 + x + 1} > 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+4}}{-2} = \frac{-1 \pm \sqrt{5}}{-2}$$

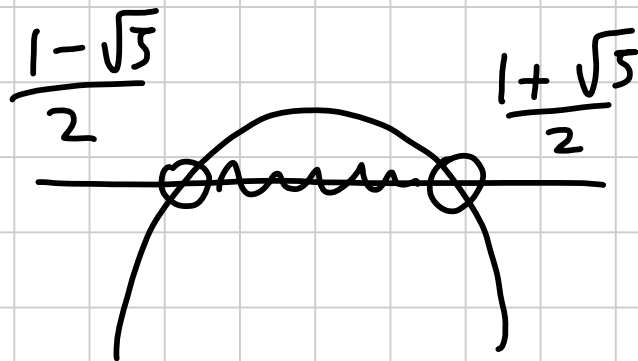
$$x_1 = \frac{-1 + \sqrt{5}}{-2} = \frac{1 - \sqrt{5}}{2}$$

|

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$$x_2 = \frac{-1 - \sqrt{5}}{-2} = \frac{1 + \sqrt{5}}{2}$$

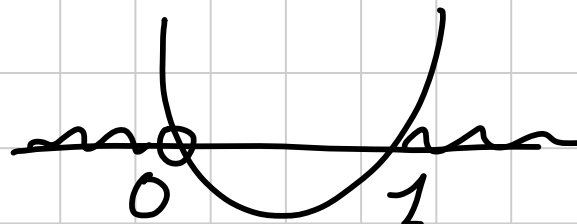


$$-0.61 \qquad 1.61$$

$$\frac{1-\sqrt{5}}{2} < x < \frac{1+\sqrt{5}}{2}$$

$$D > 0$$

$$\begin{aligned} x^2 - x &> 0 \\ x(x-1) &= 0 \\ x &= 0 \vee x = 1 \end{aligned}$$



|         |                        |   |                    |                        |
|---------|------------------------|---|--------------------|------------------------|
|         | $\frac{1-\sqrt{5}}{2}$ | 0 | $x < 0 \vee x > 1$ | $\frac{1+\sqrt{5}}{2}$ |
| N       | -                      | + | /                  | +                      |
| D       | +                      | + | /                  | +                      |
| $f'(x)$ | -                      | + | /                  | -                      |

$f(x)$

