$$\lim_{x \to 0^+} \left(\underbrace{\frac{\sqrt{x}}{e^x}}_{0} + \underbrace{\frac{x+1}{\sqrt{x}}}_{+\infty} \right) = +\infty$$

$\sqrt{x} \to 0^+$, $e^x \to 1^+$, $x+1 \to 1^+$, $\sqrt{x} \to 0^+$

$$f(x) = (3x+4)^2 \, e^{3x+4}$$

$$g(x) = (3x+4)^2$$

$$h(x) = e^{3x+4}$$

$$f(x) = g(x) \cdot h(x)$$

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

$$g(x) = (3x+4)^2$$

$$x \xrightarrow{f} 3x+4 = t \xrightarrow{m} t^2$$

$$\cancel{g} \quad g'(x) = \frac{d}{dx}(3x+4) \cdot \frac{d}{dt}(t^2)$$

$$= 3 \cdot 2t = 6t = 6(3x+4)$$

$$x \xrightarrow{\substack{(3x+4)^2 \\ f(x)}} 3x+4 = t \xrightarrow{g(t)} t^2 = (3x+4)^2$$

$$(3x+4)^2 = g(f(x))$$

$$\cancel{4}\, g(f(x))' = \frac{df}{dx} \cdot \frac{dg(f(x))}{d(f(x))} \qquad f(x) = 3x+4$$

$$\frac{df}{dx} = 3 \qquad \Rightarrow \quad (\underbrace{3x+4})^2 = z^2$$

$$g'(x) = 3 \cdot 2(3x+4)$$

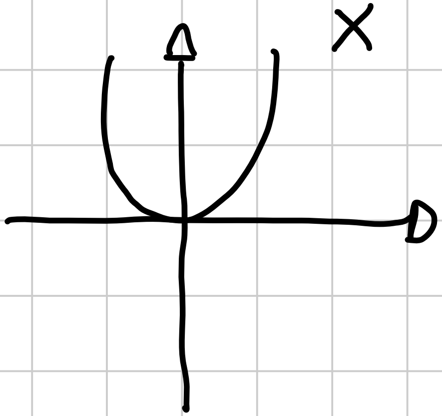
$$h'(x) = 3e^{3x+4} \qquad h(x) = e^{3x+4}$$

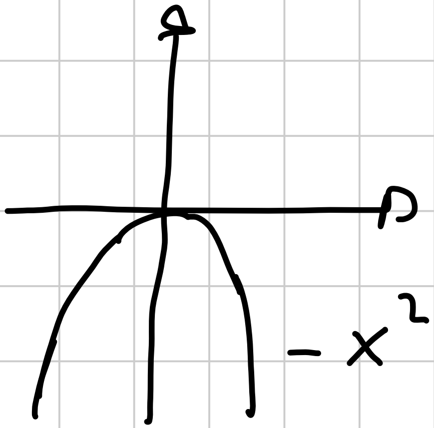
$$x \longrightarrow 3x+4 = t \longrightarrow e^t$$

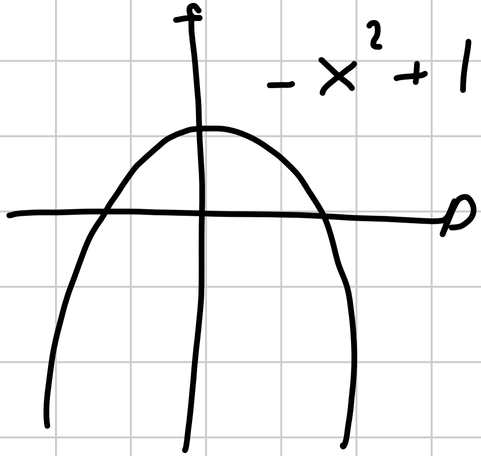
$$\frac{d(3x+4)}{dx} \cdot \frac{de^t}{dt}$$

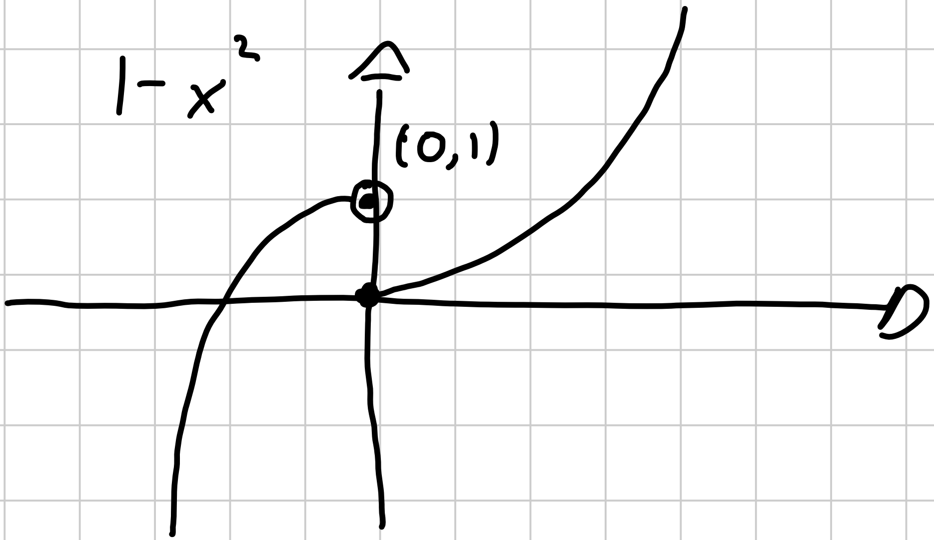
$$f'(x) = 6(3x+4)e^{3x+4} + (3x+4)^2 \cdot 3e^{3x+4}$$

$$f : \mathbb{R} \to \mathbb{R} \qquad f(x) = \begin{cases} 1 - x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$$









I specie

$$\lim_{x \to 0^-} f(x) \;/\; \lim_{x \to 0^+} f(x) \;/\; \underline{f(0) = 0}$$

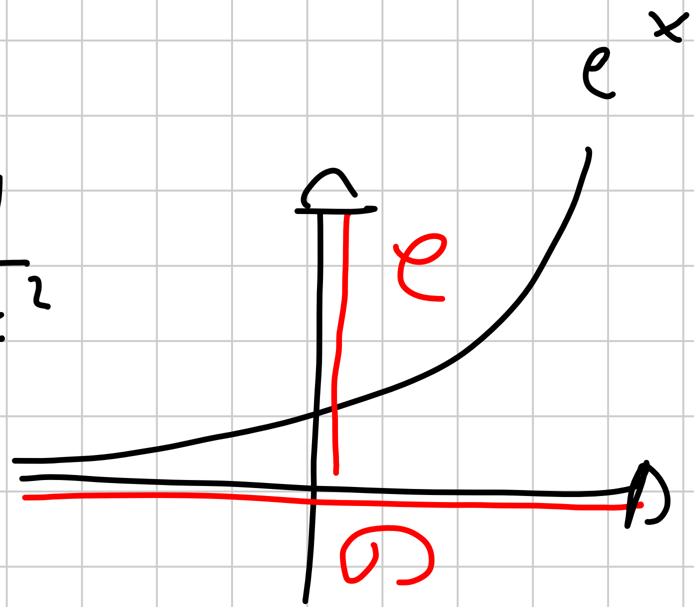
$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (1 - x^2) = 1$$

$$\lim_{x \to 0^+} x^2 = 0$$

13/11/11

ESERCIZIO 3

$f(x) = e^{-x}$ $\qquad$ $g(x) = \dfrac{1}{x^2}$



$f(g(x))$ $\qquad$ $g(f(x))$

$f(g(x))$

$$x \xrightarrow{g(x)} \frac{1}{x^2} = t \xrightarrow{f(t)} e^{-t} = e^{-\frac{1}{x^2}}$$

$D: x \neq 0$

$?\ C: \cancel{y} > 0$

$$\frac{}{?}$$

$$g(f(x))$$

$$x \xrightarrow{f(x)} e^{-x} = z \xrightarrow{g(z)} \frac{1}{z^2} = \frac{1}{(e^{-x})^2} = \frac{1}{e^{-2x}}$$

$$\lim_{x \to +\infty} \frac{f(x)}{g(x)} = 0$$

$$\lim_{x \to +\infty} \frac{e^{-x}}{\frac{1}{x^2}} = \qquad = \lim_{x \to +\infty} \underbrace{e^{-x}}_{\to 0} \cdot \underbrace{x^2}_{\to +\infty} = \lim_{x \to +\infty} \frac{x^2}{e^x} =$$

$e^{-x} \to 0$, $\frac{1}{x^2} \to 0$

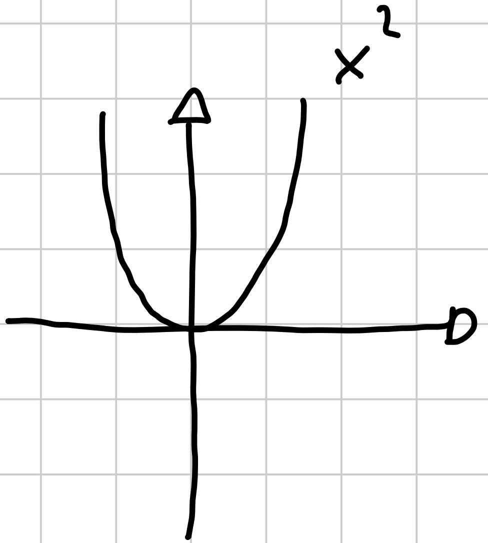
0 $\quad$ +∞ $\quad$ ∞

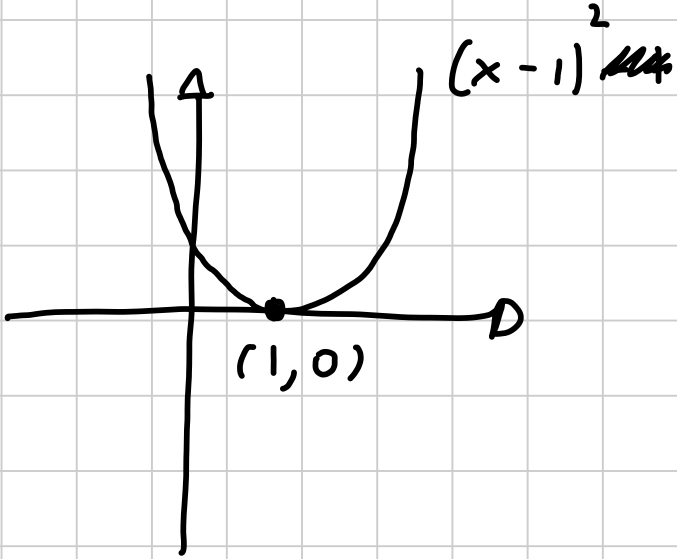
0 $\quad$ 00

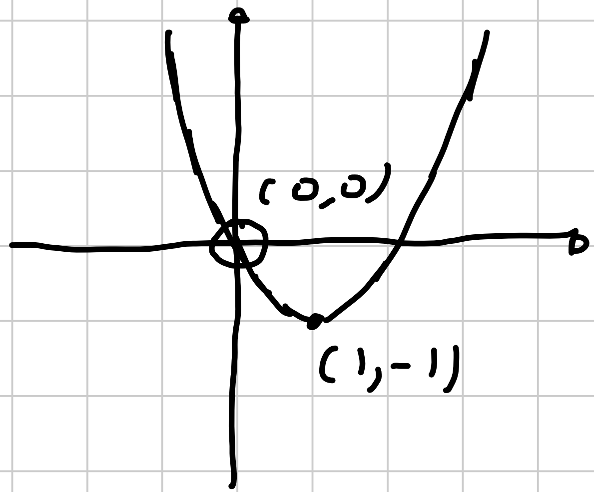
14 - 01 - 20

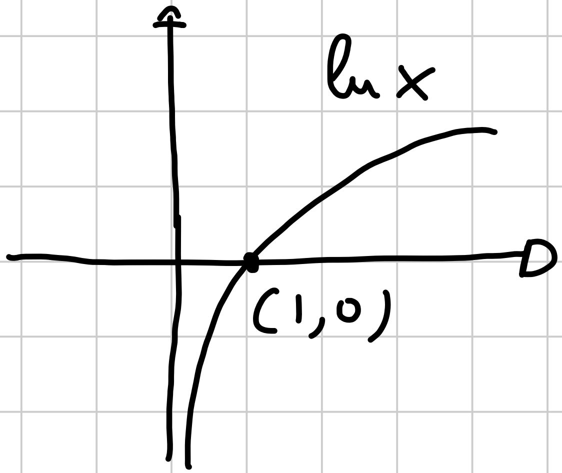
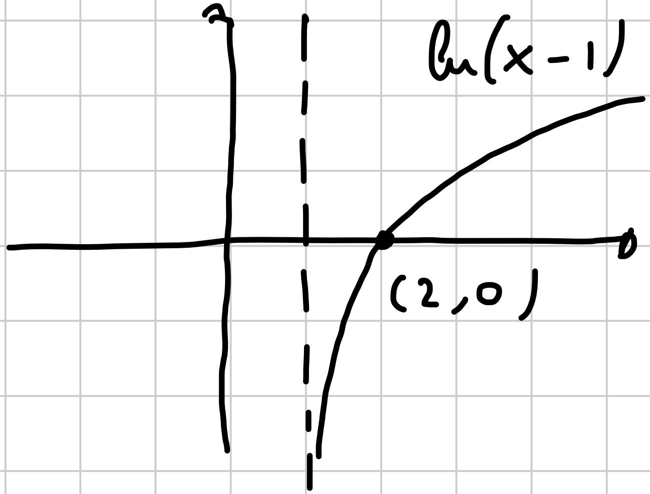
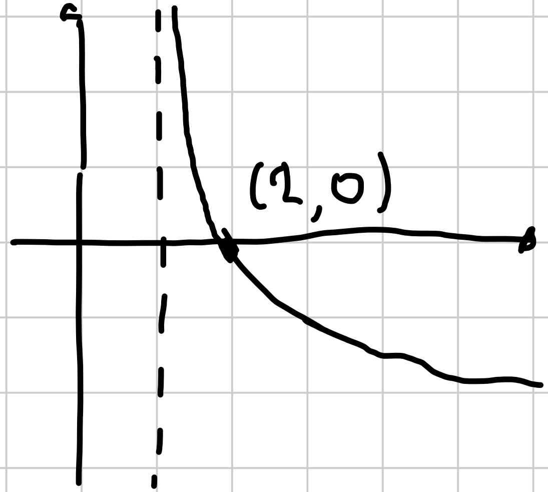
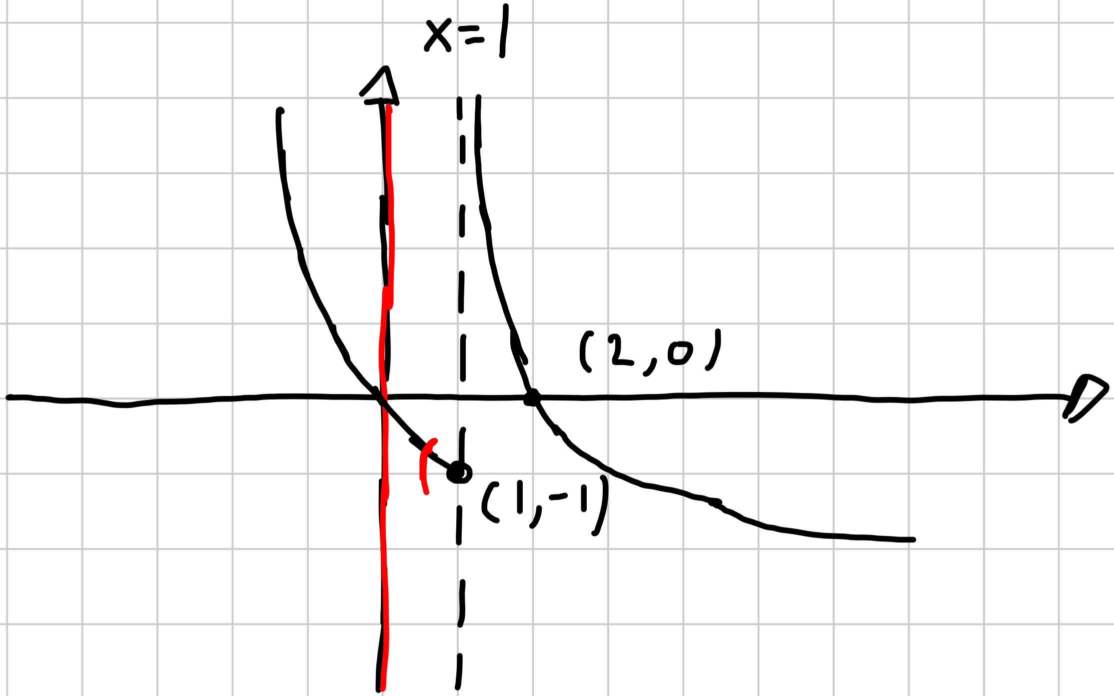
ESERCIZIO 1

$$f(x) = \begin{cases} (x-1)^2 - 1 & x \leq 1 \\ -\ln(x-1) & x > 1 \end{cases}$$








ln x
(1,0)

ln(x-1)
(2,0)

(2,0)

x=1
(2,0)
(1,-1)

derivabile $\Rightarrow$ continua

continua $\not\Rightarrow$ derivabile

non continua $\Rightarrow$ non derivabile

17/09/2013

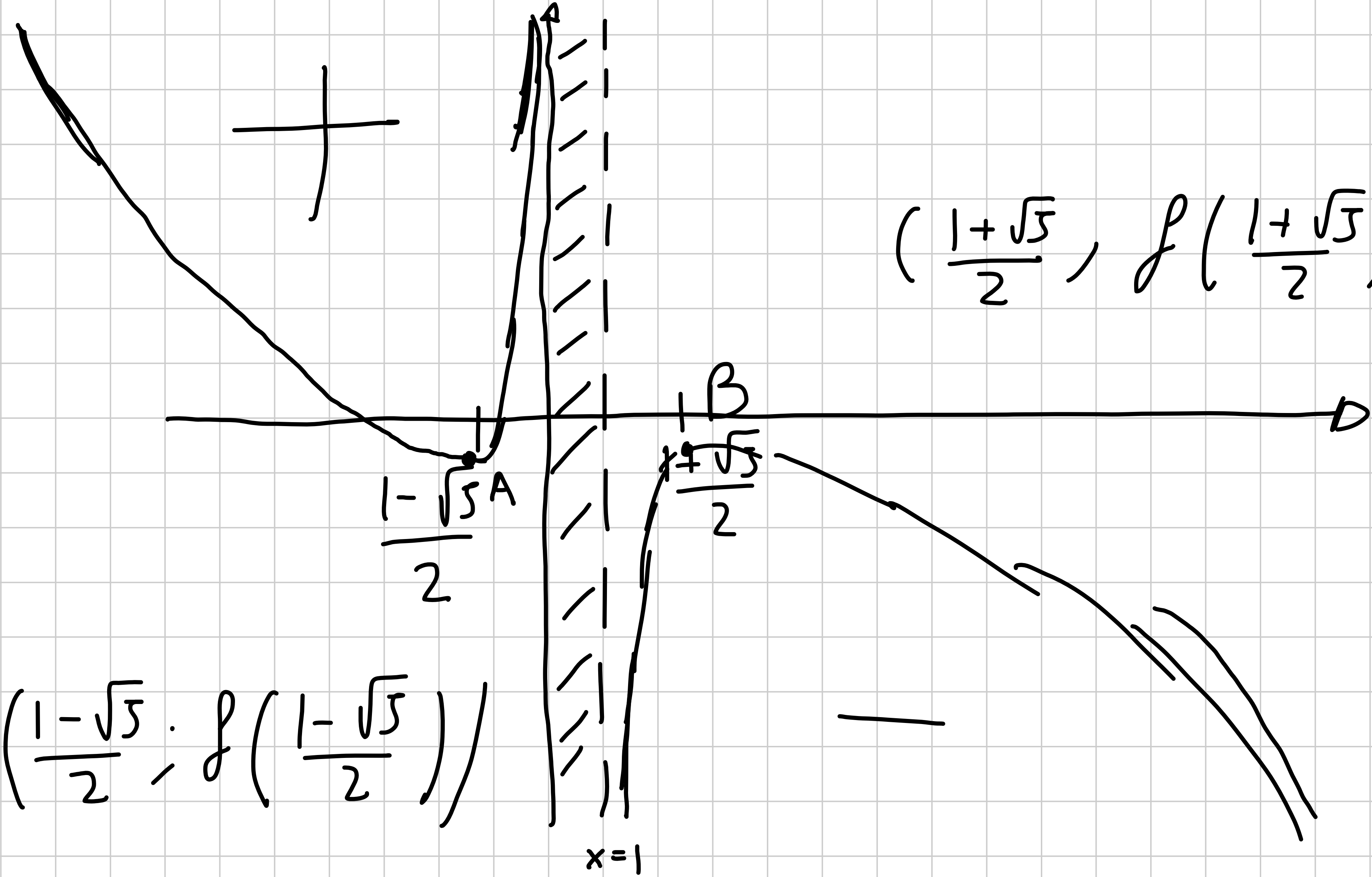
ESERCIZIO 1

$$f(x) = \ln\left(\frac{x-1}{x}\right) - x$$

$$D: \quad \frac{x-1}{x} > 0 \qquad N > 0 \quad x > 1 \qquad D > 0 \quad x > 0$$

	0		1	
$-$		$-$		$+$
$-$		$+$		$+$
$+$		$-$		$+$

$x < 0 \ \vee \ x > 1$



$$\left(\frac{1+\sqrt{5}}{2} , f\left(\frac{1+\sqrt{5}}{2} \right) \right)$$

$$A\left(\frac{1-\sqrt{5}}{2} ; f\left(\frac{1-\sqrt{5}}{2} \right) \right)$$

$$\lim_{x \to -\infty} \ln\left(\frac{\overset{\infty}{x-1}}{\underset{\infty}{x}}\right) - x = \lim_{x \to -\infty} \ln\left(\frac{x}{x}\right) - x =$$

$$= \lim_{x \to -\infty} \underbrace{\ln 1}_{0} - \underbrace{x}_{+\infty} = +\infty$$

$$\underbrace{\lim_{x \to 0^-} \ln\left(\frac{\overbrace{x-1}^{-1^-}}{\underbrace{x}_{0^-}}\right)}_{\underbrace{}_{+\infty}} - \underbrace{x}_{0^-} = +\infty \qquad -0,1-1 = -1,1$$

$$+\infty$$

$$\lim_{x \to 1^+} \ln\left(\frac{\overbrace{x-1}^{\to 0^+}}{\underbrace{x}_{\to 1^+}}\right) - \underbrace{x}_{1^+} = -\infty \qquad 1,1-1 = 0,1$$

$$\underbrace{\underbrace{}_{0^+}}_{-\infty}$$

$$\lim_{x \to +\infty} \ln\left(\frac{x-1}{x}\right) - x = \lim_{x \to +\infty} -x = -\infty$$

$$f(x) = \ln\left(\frac{x-1}{x}\right) - x$$

Intersezioni asse x

$$f(x) = 0 \rightarrow \ln\left(\frac{x-1}{x}\right) - x = 0$$

DERIVATA PRIMA

$$f(x) = \ln\left(\frac{x-1}{x}\right) - x$$

$$D\,f(x) = D\left[\ln\left(\frac{x-1}{x}\right)\right] - D[x]$$

$$D\left[\ln\left(\frac{x-1}{x}\right)\right] = D\left[\frac{x-1}{x}\right] \cdot D[\ln t]$$

$$x \xrightarrow{g(x)} \frac{x-1}{x} = t \xrightarrow{h(t)} \ln t$$

$$D\left[\frac{x-1}{x}\right] = \frac{1x-(x-1)}{x^2} = +\frac{1}{x^2}$$

$$D[\ln t] = \frac{1}{t} = \frac{1}{\frac{x-1}{x}} = \frac{x}{x-1}$$

$$D\left[\ln\left(\frac{x-1}{x}\right)\right] = +\frac{1}{x^2}\cdot\frac{x}{x-1} = +\frac{1}{x(x-1)}$$

$$D\left[\ln\left(\frac{x-1}{x}\right) - x\right] = +\frac{1}{x(x-1)} - 1$$

$$f'(x) > 0$$

$$\frac{1}{x(x-1)} - 1 > 0$$

$$\frac{1 - x(x-1)}{x(x-1)} > 0$$

$$\frac{1 - x^2 + x}{x^2 - x} > 0 \longrightarrow \frac{-x^2 + x + 1}{x^2 - x} > 0$$

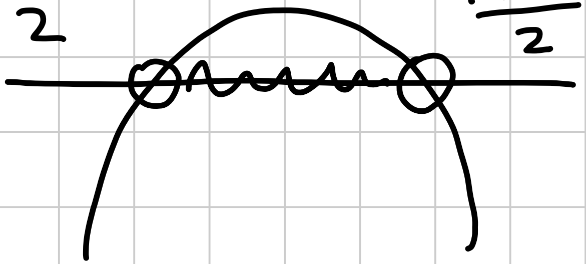
$$N > 0 \qquad \overline{-x^2 + x + 1 > 0}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+4}}{-2} = \frac{-1 \pm \sqrt{5}}{-2}$$

$$x_1 = \frac{-1 + \sqrt{5}}{-2} = \frac{1 - \sqrt{5}}{2}$$

$$x_2 = \frac{-1-\sqrt{5}}{-2} = \frac{1+\sqrt{5}}{2}$$

$\frac{1-\sqrt{5}}{2}$ $\qquad$ $\frac{1+\sqrt{5}}{2}$



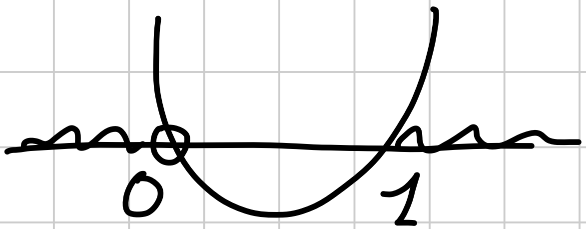
$-0.61 \qquad 1.61$

$$\frac{1-\sqrt{5}}{2} < x < \frac{1+\sqrt{5}}{2}$$

$D > 0 \qquad x^2 - x > 0$

$x(x-1) = 0$

$x = 0 \ \vee \ x = 1$



$x < 0 \ \vee \ x > 1$

		$\frac{1-\sqrt{5}}{2}$		0		1		$\frac{1+\sqrt{5}}{2}$	
N	$-$		$+$		//////		$+$		$-$
D	$+$		$+$		//////		$+$		$+$
$f'(x)$	$-$		$+$		//////		$+$		$-$

$f(x) \quad \searrow \;\big|\; \nearrow \;\big|\;\;\big|\; \nearrow \;\big|\; \searrow$