DERIVATE

$$f(x) = x\, e^{x+\sqrt{x}}$$

$$f'(x) = \underbrace{D[x]}_{1}\, e^{x+\sqrt{x}} + x \cdot \underbrace{D\left[e^{x+\sqrt{x}}\right]}$$

$$e^{x+\sqrt{x}} = f. \text{ composta}$$

$$f(x) = x+\sqrt{x}$$

$$g(f(x)) = e^{x+\sqrt{x}}$$

$$x \longrightarrow x+\sqrt{x} = t \longrightarrow e^{t}$$

$$D[x+\sqrt{x}] \cdot D[e^{t}] =$$

$$D\left[x^{\frac{1}{2}}\right] = \frac{1}{2} x^{\frac{1}{2}-1}$$

$$D[x^5] = 5x^{5-1}$$

$$D[x+\sqrt{x}] = D[x] + D[\sqrt{x}] = 1 + \frac{1}{2} x^{-\frac{1}{2}}$$

$$\Big| = 1 + \frac{1}{2\sqrt{x}}$$

$$D[e^t] = e^t = e^{x+\sqrt{x}}$$

$$D[x+\sqrt{x}] \cdot D[e^t] = \left(1 + \frac{1}{2\sqrt{x}}\right) e^{x+\sqrt{x}}$$

$$f'(x) = e^{x+\sqrt{x}} + x\left(1 + \frac{1}{2\sqrt{x}}\right) e^{x+\sqrt{x}}$$

$$= \left(1 + x\left(1 + \frac{1}{2\sqrt{x}}\right)\right) e^{x+\sqrt{x}}$$

$$f(x) = \sqrt[3]{x}\, e^{x-x^2}$$

$$f'(x) = D[\sqrt[3]{x}]\,e^{x-x^2} + \sqrt[3]{x}\,D[e^{x-x^2}]$$

$$D[\sqrt[3]{x}] = D[x^{\frac{1}{3}}] = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}} =$$

$$D[x^{\alpha}] = \alpha x^{\alpha-1}$$

$$= \frac{1}{3\sqrt[3]{x^2}}$$

$$D[e^{x-x^2}] = D[x-x^2]\cdot D[e^t] = (*)$$

$$f(x) = x - x^2$$

$$g(f(x)) = e^{x-x^2}$$

$$\cancel{x} \longrightarrow \underbrace{x-x^2}_{} = t \longrightarrow \underbrace{e^t}_{}$$

$$(*) = (1-2x)e^{t} = (1-2x)e^{x-x^2}$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}\, e^{x-x^2} + \sqrt[3]{x}\,(1-2x)\,e^{x-x^2} =$$

$$CE: \begin{cases} x > 0 \\ \frac{1}{x} + \ln x \geq 0 \end{cases}$$

$$f(x) = \left(\frac{1}{x} + \ln x\right)^{\frac{3}{5}} = \sqrt[4]{\left(\frac{1}{x} + \ln x\right)^3}$$

$$f(x) = \frac{1}{x} + \ln x$$

$$g(f(x)) = f(x)^{\frac{3}{5}}$$

$$x \longrightarrow \frac{1}{x} + \ln x = t \longrightarrow t^{\frac{3}{5}}$$

$$f'(x) = D\left[\frac{1}{x} + \ln x\right] \cdot D\left[t^{\frac{3}{4}}\right] = \left(-\frac{1}{x^2} + \frac{1}{x}\right) \cdot \frac{3}{4\sqrt[4]{\frac{1}{x} + \ln x}}$$

$$D\left[\frac{1}{x} + \ln x\right] = D\left[\frac{1}{x}\right] + D\left[\ln x\right] = -\frac{1}{x^2} + \frac{1}{x}$$

$$D\left[t^{\frac{3}{4}}\right] = \frac{3}{4} t^{\frac{3}{4} - 1} = \frac{3}{4} t^{\frac{3}{4} - \frac{4}{4}} = \frac{3}{4} t^{-\frac{1}{4}} = \frac{3}{4t^{\frac{1}{4}}} = \frac{3}{4\sqrt[4]{t}} =$$

$$= \frac{3}{4\sqrt[4]{\frac{1}{x} + \ln x}}$$

$$f(x) = \ln\left(\sqrt{x^2 + 2}\right)$$

$$x \longrightarrow \underbrace{x^2 + 2}_{} = t \longrightarrow \underbrace{\sqrt{t}}_{} = z \longrightarrow \underbrace{\ln z}_{}$$

$$f'(x) = D[x^2+2] \cdot D[\sqrt{t}] \cdot D[\ln z] =$$

$$= 2x \cdot \frac{1}{2\sqrt{t}} \cdot \frac{1}{z} = 2x \cdot \frac{1}{2\sqrt{x^2+2}} \cdot \frac{1}{\sqrt{t}} =$$

$$= \cancel{2}x \cdot \frac{1}{\cancel{2}\sqrt{x^2+2}} \cdot \frac{1}{\sqrt{x^2+2}} = \frac{x}{x^2+2}$$

$$\cancel{\sqrt{x^2+2} \cdot \sqrt{x^2+2}}$$

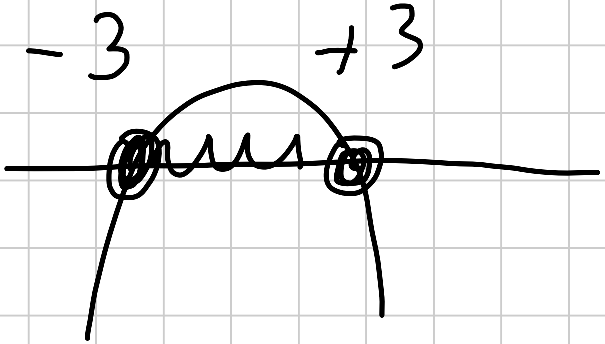
CONTINUITÀ e DERIVABILITÀ

$$f(x) = \begin{cases} 9 - x^2 & \boxed{9 - x^2 \geq 0} \\ -9 + x^2 & \boxed{9 - x^2 < 0} \end{cases}$$

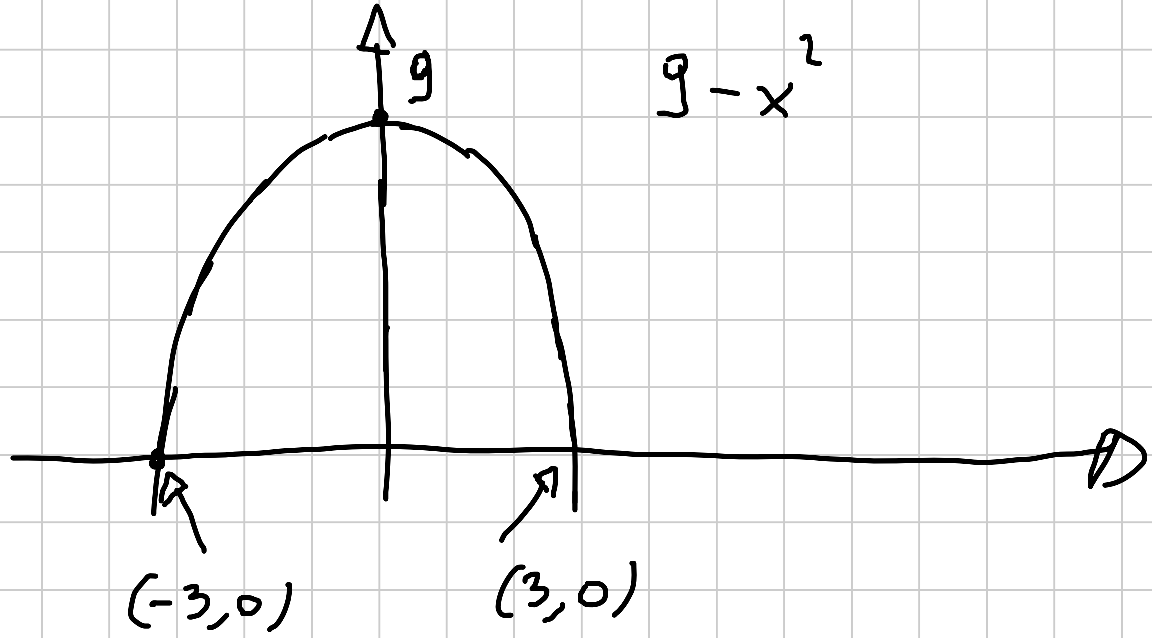
$$9 - x^2 \geq 0$$

$$+x^2 = +9$$

$$x = \pm\sqrt{9}$$

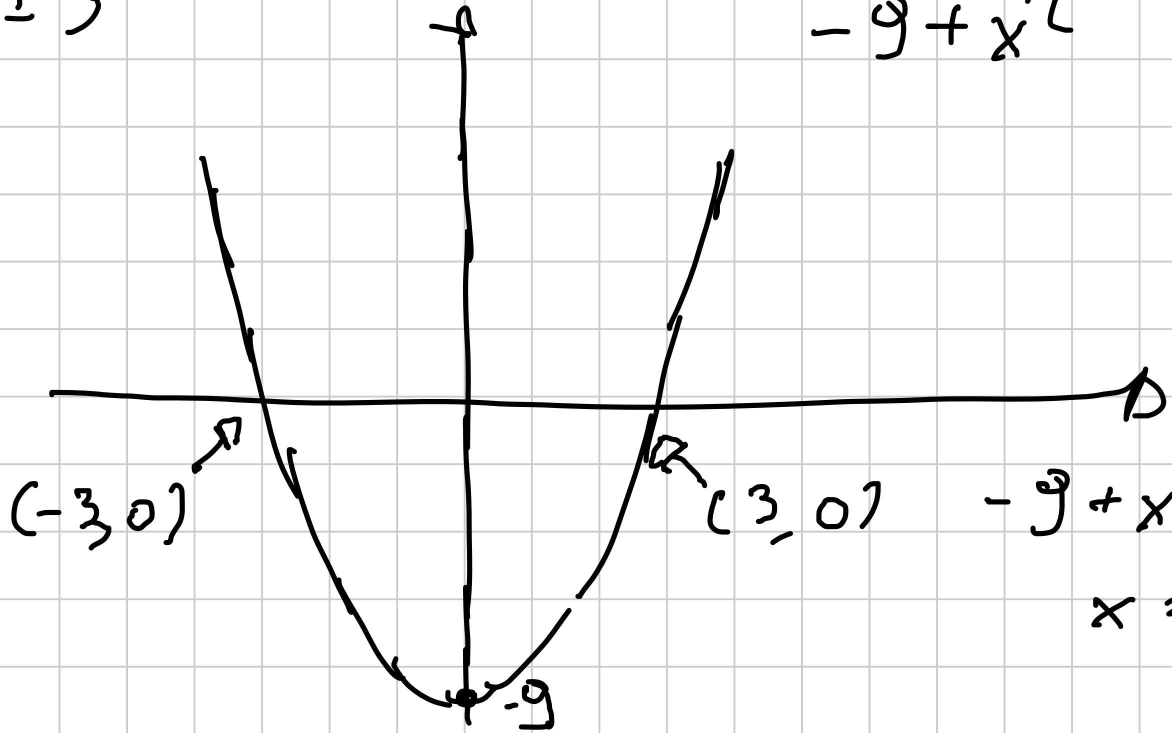


$$f(x) = \begin{cases} 9 - x^2 & -3 \leq x \leq 3 \\ -9 + x^2 & x < -3 \ \vee \ x > 3 \end{cases}$$



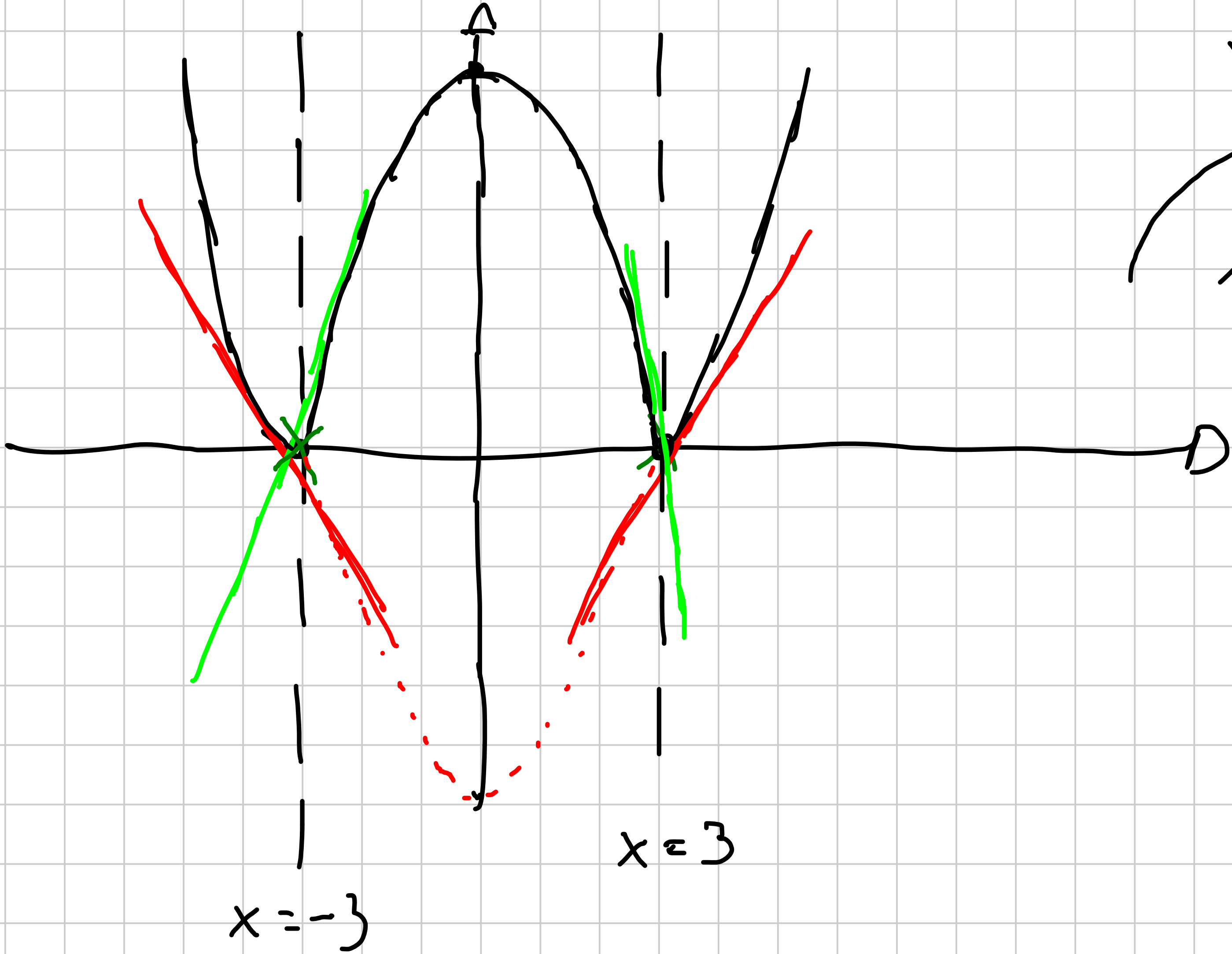
$$9 - x^2 = 0$$

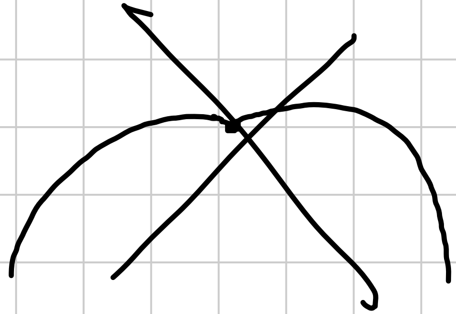
$$x = \pm 3$$



$$-9 + x^2 = 0$$

$$x = \pm 3$$


x = 3
x = -3



$$f'(x) = \begin{cases} -2x & -3 < x < 3 \\ 2x & x < -3 \vee x > 3 \end{cases}$$

$x = -3 \qquad x = 3$

$$\lim_{x \to -3^-} f'(x) = \lim_{x \to -3^-} 2x = -6$$

$$\lim_{x \to -3^+} f'(x) = \lim_{x \to -3^+} -2x = +6$$

$x = -3 \wedge x = 3$ non derivabile

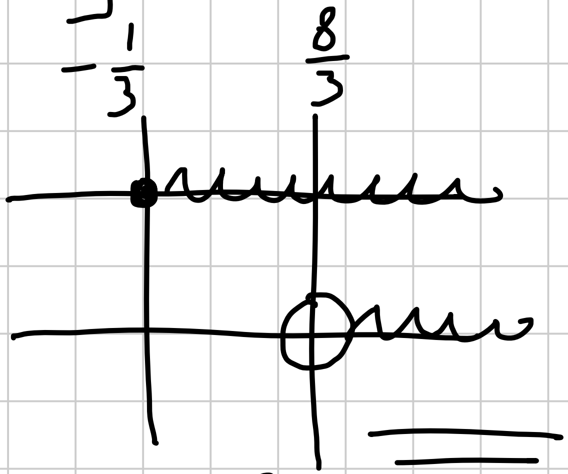
$$\lim_{x \to 3^-} f'(x) = \lim_{x \to 3^-} -2x = -6$$

$$\lim_{x \to 3^+} f'(x) = \lim_{x \to 3^+} 2x = 6$$

$$\sqrt{3x+1} > 3$$

$$\begin{cases} 3x+1 \geq 0 \\ \left(\sqrt{3x+1}\right)^2 > 3^2 \end{cases} \qquad \begin{cases} x \geq -\frac{1}{3} \\ 3x+1 > 9 \end{cases}$$

$$b) \begin{cases} x \geq -\frac{1}{3} \\ 3x-8 > 0 \end{cases} \qquad \begin{cases} x \geq -\frac{1}{3} \\ x > \frac{8}{3} \end{cases}$$



$$x > \frac{8}{3}$$

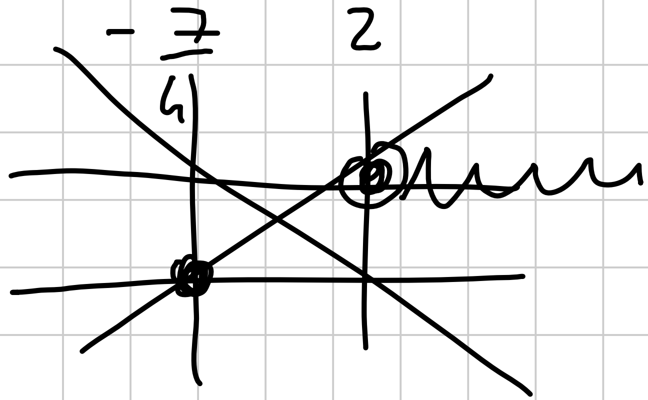
$$b) \qquad = 0$$

$\underbrace{\sqrt{x+9}} < -3$ nessuna soluzione

$$1 - 2\cdot\sqrt{x+2} = 0$$

$$-2\sqrt{x+2} = -1$$

$$2\sqrt{x+2} = 1$$



$$\begin{cases} x+2 \geq 0 \\ \left(2\sqrt{x+2}\right)^2 = 1^2 \end{cases} \quad \begin{cases} x \geq -2 \\ 2^2 \cdot \left(\sqrt{x+2}\right)^2 = 1 \end{cases} \quad b) \begin{cases} x \geq -2 \\ 4(x+2) = 1 \end{cases}$$

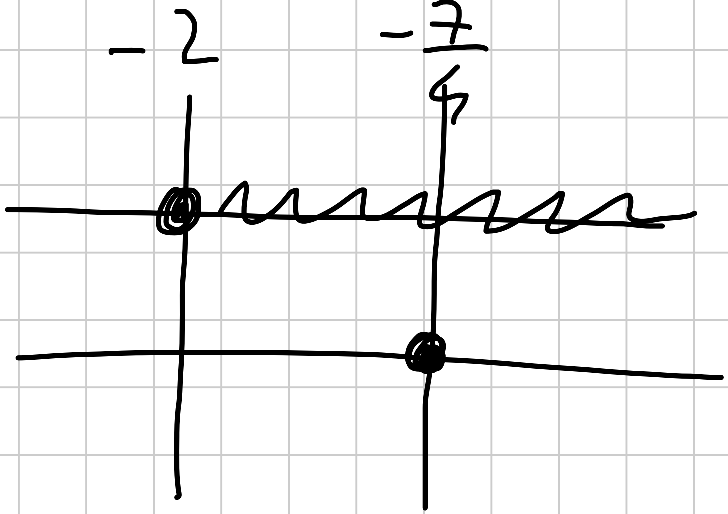
b)

$$4x + 8 = 1$$

$$4x = -7$$

$$x = -\frac{7}{4}$$

$$\begin{cases} x \geq -2 \\ x = -\frac{7}{4} \end{cases}$$



$$x = -\frac{7}{5}$$

$$f(x) = x^2 e^{-x} \qquad \forall x \in \mathbb{R}$$

$$\lim_{x \to -\infty} x^2 e^{-x} = +\infty$$

$+\infty \qquad +\infty$

$$\lim_{x \to +\infty} x^2 e^{-x} = \lim_{x \to +\infty} \frac{x^2}{e^x} = 0$$

$+\infty \cdot 0$

INTERSEZIONE ASSE Y $\quad (0,0)$

$$f(0) = 0^2 e^0 = 0 \cdot 1 = 0$$

INTERSEZIONE ASSE X : $f(x) = 0 \qquad x^2 e^{-x} = 0 \quad (0,0)$

$+$

$$\cancel{\boxed{x + \ln x = 0}}$$

$$\cancel{x^2 + 2x + 1 = 0}$$

$$f'(x) = D[x^2]\,e^{-x} + x^2 \cdot D[e^{-x}]$$

$$= 2x\,e^{-x} + x^2(-e^{-x}) = 2x\,e^{-x} - x^2 e^{-x}$$

$$= (2x - x^2)\,e^{-x}$$

$$D[e^{-x}] = -e^{-x}$$

$$x \longrightarrow -x = t \longrightarrow e^{t}$$

$$D[-x] \cdot D[e^{t}] = -1\,e^{t} = -1\,e^{-x} = -e^{-x}$$

$f'(x) \geq 0 \qquad \underbrace{(2x - x^2)}\, e^{-x} \geq 0 \qquad 0 \leq x \leq 2$

$e^{-x} \Rightarrow +$

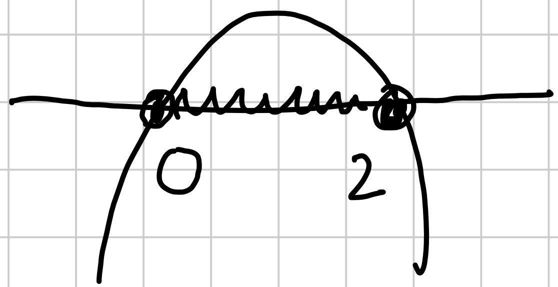
$$2x - x^2 \geq 0$$

$$2x - x^2 = 0$$

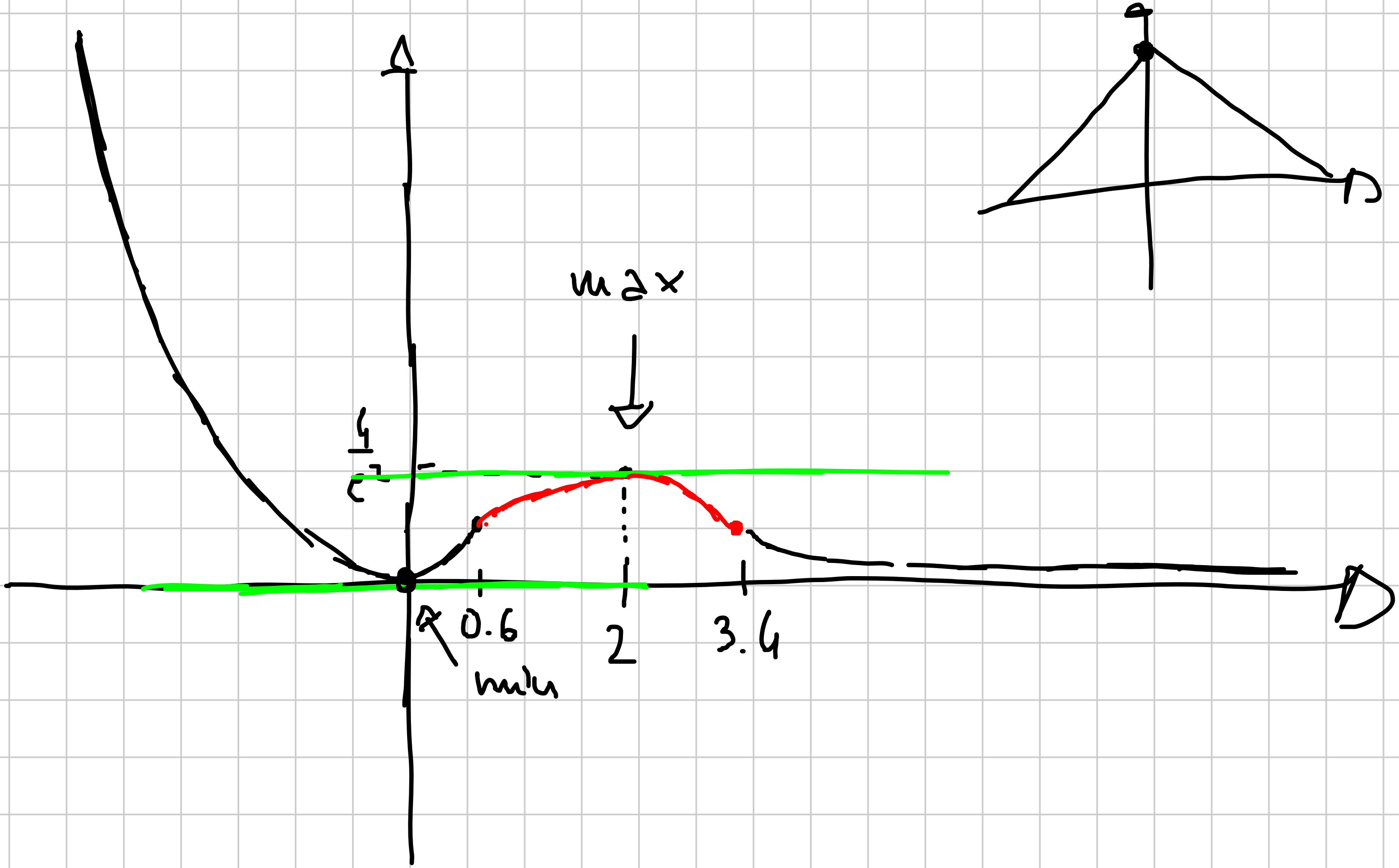
$$x(-x+2) = 0$$

$$x = 0$$

$$x = 2$$



		0		2	
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$
		min		max	



min $\quad x = 0 \quad f(0) = 0$

max $\quad x = 2 \quad f(2) = 2^2 e^{-2} = \frac{4}{e^2}$

$$f'(x) = (2x - x^2)e^{-x}$$

$$f''(x) = D[2x - x^2]e^{-x} + (2x - x^2)D[e^{-x}] =$$

$$= (2 - 2x)e^{-x} + (2x - x^2)(-e^{-x})$$

$$= (2 - 2x)e^{-x} - (2x - x^2)e^{-x}$$

$$= (2 - 2x - 2x + x^2)e^{-x}$$

$$= (x^2 - 4x + 2)e^{-x}$$

$$f''(x) \geq 0 \qquad (x^2 - 4x + 2)e^{-x} \geq 0$$

$+$

$$x^2 - 4x + 2 \geq 0$$

$$x \leq 2 - \sqrt{2} \quad \vee \quad x \geq 2 + \sqrt{2}$$

		$2-\sqrt{2}$		$2+\sqrt{2}$	
f''	$+$		$-$		$+$
f	$\cup$		$\cap$		$\cup$
		0.6		3.4	