$$\int x^{\alpha}\,dx = \frac{x^{\alpha+1}}{\alpha+1} + c$$

$$\int \frac{1}{x}\,dx = \ln|x| + c \qquad \int e^{x}\,dx = e^{x} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\,e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int [f(x)]^{k} f'(x)\,dx = \frac{f(x)^{k+1}}{k+1} + c$$

1) $\int \left(\frac{1}{2}x^3 + \frac{1}{3}x^2 - \frac{1}{4}x - \frac{2}{3} \right) dx =$ $\int k f(x) = k \int f(x)$

$\int \frac{1}{2}x^3\,dx + \int \frac{1}{3}x^2\,dx - \int \frac{1}{4}x\,dx - \int \frac{2}{3}\,dx$

$\frac{1}{2}\int x^3\,dx + \frac{1}{3}\int x^2\,dx - \frac{1}{4}\int x\,dx - \frac{2}{3}\int 1\,dx$

$\frac{1}{2}\,\frac{x^4}{4} + \underbrace{\frac{1}{3}\cdot\frac{x^3}{3}} - \frac{1}{4}\,\frac{x^2}{2} - \frac{2}{3}\cdot x + \cancel{c} =$

$\frac{x^4}{8} + \frac{x^3}{9} - \frac{x^2}{8} - \frac{2}{3}x + \cancel{c}$

2) $$\int\left(\sqrt[3]{x^2}+\sqrt[4]{x^3}\right)dx = \int\sqrt[3]{x^2}\,dx + \int\sqrt[4]{x^3}\,dx =$$

$$= \int x^{\frac{2}{3}}\,dx + \int x^{\frac{3}{4}}\,dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + \frac{x^{\frac{3}{4}+1}}{\frac{3}{4}+1} + C$$

$$\int x^{\alpha}\,dx = \frac{x^{\alpha+1}}{\alpha+1}$$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{x^{\frac{7}{4}}}{\frac{7}{4}} = \frac{3}{5}\sqrt[3]{x^5} + \frac{4}{7}\sqrt[4]{x^7} + C$$

3) $$\int \frac{2\sqrt{x}-\sqrt[3]{x}}{x}\,dx = \int\frac{2\sqrt{x}}{x}\,dx - \int\frac{\sqrt[3]{x}}{x}\,dx =$$

$$\int x^{\alpha}\,dx$$

$$= 2\int \frac{x^{\frac{1}{2}}}{x}\,dx - \int \frac{x^{\frac{1}{3}}}{x}\,dx = 2\int x^{\frac{1}{2}-1}\,dx - \int x^{\frac{1}{3}-1}\,dx$$

$$= 2\int x^{\frac{1}{2}-\frac{2}{2}}\,dx - \int x^{\frac{1}{3}-\frac{3}{3}}\,dx = 2\int x^{-\frac{1}{2}}\,dx - \int x^{-\frac{2}{3}}\,dx =$$

$$= 2\,\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + \cancel{C} = 2\,\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{1}{3}}}{\frac{1}{3}} =$$

$$= 2\cdot 2\sqrt{x} - 3\sqrt[3]{x} = 4\sqrt{x} - 3\sqrt[3]{x}$$

4) $\int \sqrt{x+2}\,dx = \int 1(x+2)^{\frac{1}{2}}\,dx = \frac{(x+2)^{\frac{1}{2}+1}}{\frac{1}{2}+1} =$

$$\int f'(x)\cdot f(x)^{k} = \frac{f(x)^{k+1}}{k+1}$$

$$= \frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}\sqrt[2]{(x+2)^3} + C$$

$$\int \sqrt{x+2}\,dx = \int \sqrt{y}\,dy = \int y^{\frac{1}{2}}\,dy =$$

c.v $x+2 = y$

$1dx = 1dy$

$dx = dy$

$$= \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{y^{\frac{3}{2}}}{\frac{3}{2}} =$$

$$= \frac{2}{3}\sqrt[2]{y^3} = \frac{2}{3}\sqrt{(x+2)^3} + C$$

$$\frac{1}{(x-3)^{\frac{1}{2}}}$$

5) $\int \frac{1}{\sqrt{x-3}}\,dx = \int 1(x-3)^{-\frac{1}{2}}\,dx = \int f'(x)\cdot f(x)^{-\frac{1}{2}}\,dx =$

$$= \frac{f(x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{(x-3)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{(x-3)^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{x-3} + c'$$

C.V. $x - 3 = y$

$dx = dy$

$D[3-2x]$

$$6) \int \sqrt{3-2x}\, dx = \left(-\frac{1}{2}\right) \int -2\,(3-2x)^{\frac{1}{2}}\, dx = -\frac{1}{2} \int f'(x)\, f(x)^{\frac{1}{2}}\, dx =$$

$$= -\frac{1}{2} \cdot \frac{f(x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} = -\frac{1}{2}\, \frac{(3-2x)^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{1}{2} \cdot \frac{2}{3} \sqrt{(3-2x)^3} =$$

$$= -\frac{1}{3} \sqrt{(3-2x)^3} + c$$

$$\int \sqrt{3-2x}\, dx =$$

t

$$3-2x = t \qquad\qquad 3-2x = t$$

$$-2dx = 1dt \qquad\qquad D[3-2x]dx = D[t]dt$$

$$-2dx = dt \qquad \longrightarrow \quad -2dx = dt \quad \longleftarrow$$

$$dx = \frac{dt}{-2}$$

$$dx = -\frac{1}{2}dt$$

$$\int \sqrt{t}\left(-\frac{1}{2}dt\right) = -\frac{1}{2}\int \sqrt{t}\, dt = -\frac{1}{2}\int t^{\frac{1}{2}}\, dt =$$

$$= -\frac{1}{2}\,\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} = -\frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{1}{2}\cdot\frac{2}{3}\sqrt{t^3} =$$

$$= -\frac{1}{3}\sqrt{(3-2x)^3} + C$$

7) $$\int 1 e^{1-x}\, dx =$$

$$f(x) = 1-x$$

$$f'(x) = -1$$

$$\int f'(x)\, e^{f(x)}\, dx = e^{f(x)} + c$$

$$-1 \int -1 e^{1-x}\, dx = -1 \int \underbrace{f'(x)}_{-1}\, e^{\overbrace{f(x)}^{1-x}}\, dx =$$

$$-1\, e^{f(x)} = -e^{1-x} + c$$

$$\int e^{1-x}\, dx = \int e^{y}(-dy) = -\int e^{y}\, dy =$$

$$1-x = y$$

$$-1\, dx = dy$$

$$= -e^{y} = -e^{1-x} + c$$

$$dx = -dy$$

•) $- \int \frac{-e^x}{1-e^x}\, dx = -\int \frac{-e^x}{1-e^x}\, dx = -\underbrace{\int \frac{f'(x)}{f(x)}\, dx}$

$$D\left[1-e^x\right] = -e^x$$

$$= -\ln|f(x)| = -\ln|1-e^x| + c$$

•) $\frac{1}{4}\int 4x\, e^{1+2x^2}\, dx$

$$\int f'(x)\, e^{f(x)}\, dx$$

$$f(x) = 1 + 2x^2$$

$$f'(x) = 4x$$

$$= \frac{1}{4} \int f'(x)\, e^{f(x)}\, dx = \frac{1}{4} e^{f(x)} = \frac{1}{4} e^{1+2x^2} + C$$

•) $$\int x\sqrt{1+3x^2}\, dx = \frac{1}{6}\int 6x\,(1+3x^2)^{\frac{1}{2}}\, dx =$$

$$f(x) = 1 + 3x^2$$
$$f'(x) = 6x$$

$$= \frac{1}{6}\int f'(x)\cdot f(x)^{\frac{1}{2}}\, dx = \frac{1}{6}\,\frac{f(x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} =$$

$$= \frac{1}{6}\,\frac{(1+3x^2)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{6}\cdot\frac{2}{3}\sqrt{(1+3x^2)^3} + C$$

$$\int x\sqrt{1+3x^2}\,dx = \int \underbrace{\sqrt{1+3x^2}}_{\sqrt{y}}\ \underbrace{x\,dx}_{\frac{dy}{6}} =$$

$$1+3x^2 = y$$
$$\longrightarrow 6x\,dx = dy \longleftarrow$$
$$x\,dx = \frac{dy}{6}$$

$$= \int \sqrt{y}\,\frac{dy}{6} = \frac{1}{6}\int \sqrt{y}\,dy = \frac{1}{6}\int y^{\frac{1}{2}}\,dy =$$

$$= \frac{1}{6}\,\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{1}{6}\,\frac{y^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{6}\cdot\frac{2}{3}\sqrt{y^3} =$$

$$= \frac{1}{6}\cdot\frac{2}{3}\sqrt{(1+3x^2)^3} + C$$

I. per parti

$$\int D\left[f(x)g(x)\right] = \int f'(x)\,g(x) + \int f(x)\,g'(x)$$

$$f(x)g(x) = \int f'(x)\,g(x) + \int f(x)\,g'(x)$$

$$\int f'(x)\,g(x) = f(x)\,g(x) - \int f(x)\,g'(x)$$

$$\int \underbrace{1}_{f'(x)} \underbrace{\ln(2x+1)}_{g(x)}\, dx =$$

$$f'(x) = 1 \qquad f(x) = x$$

$$g(x) = \ln(2x+1) \qquad g'(x) = \frac{2}{2x+1}$$

$$= \underbrace{x}_{f} \cdot \underbrace{\ln(2x+1)}_{g} - \int \underbrace{x}_{f} \cdot \underbrace{\frac{2}{2x+1}}_{g'}\, dx =$$

$$= x \cdot \ln(2x+1) - \int \frac{2x}{2x+1}\, dx = x \ln(2x+1) - \int \left(1 - \frac{1}{2x+1}\right) dx$$

$$\boxed{\frac{2x}{2x+1} = \frac{2x+1-1}{2x+1} = \frac{(2x+1)}{2x+1} - \frac{1}{2x+1}}$$

$$\Longrightarrow \; = 1 - \frac{1}{2x+1}$$

$$\begin{array}{r|l} 2x & 2x+1 \\ \cline{2-2} -2x \;\; -1 & 1 \quad Q \\ \hline 0 \;\; -1 & \\ R & \end{array}$$

$$\frac{2x}{2x+1} = Q + \frac{R}{2x+1}$$

$$\frac{2x}{2x+1} = 1 - \frac{1}{2x+1}$$

$$= x \ln(2x+1) - \left[\int 1\,dx - \int \frac{1}{2x+1}\,dx \right]$$

$$= x\ln(2x+1) - \underbrace{\int dx}_{x} + \underbrace{\int \frac{1}{2x+1}\,dx}_{\int \frac{f'(x)}{f(x)}\,dx}$$

$$f(x) = 2x+1$$

$$f'(x) = 2$$

$$\Rightarrow \int \frac{1}{2x+1}\,dx = \frac{1}{2}\int \frac{2}{2x+1}\,dx = \frac{1}{2}\int \frac{f'(x)}{f(x)}\,dx = \frac{1}{2}\ln|f(x)|$$

$$= -\frac{1}{2}\ln|2x+1|$$

$$= x\ln(2x+1) - x + \frac{1}{2}\ln(2x+1) + C$$

•) $\int \sqrt{x^3} \ln x \, dx = \int x^{\frac{3}{2}} \ln x \, dx =$

$f'(x) = x^{\frac{3}{2}}$ $\qquad f(x) = \frac{2}{5} x^{\frac{5}{2}}$

$g(x) = \ln x$ $\qquad g'(x) = \frac{1}{x}$

$$\int x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{5} \sqrt{x^5}$$

$$= \underbrace{\frac{2}{5} x^{\frac{5}{2}}}_{f} \cdot \underbrace{\ln x}_{g} - \int \underbrace{\frac{2}{5} x^{\frac{5}{2}}}_{f} \cdot \underbrace{\frac{1}{x}}_{g'} \, dx$$

$$= \frac{2}{5} x^{\frac{5}{2}} \ln x - \frac{2}{5} \int \frac{x^{\frac{5}{2}}}{x} \, dx =$$

$$\int \frac{x^{\frac{5}{2}}}{x} \, dx = \int x^{\frac{5}{2}-1} \, dx = \int x^{\frac{3}{2}} \, dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} =$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{5} x^{\frac{5}{2}}$$

$$= \frac{2}{5} x^{\frac{5}{2}} \ln x - \frac{2}{5} \cdot \frac{2}{5} x^{\frac{5}{2}} = \frac{2}{5} \sqrt{x^5} \ln x - \frac{4}{25} \sqrt{x^5} + C$$

$$\int x e^{-x}\,dx = -xe^{-x} - \int (-e^{-x})\cdot 1\,dx =$$

$f'(x) = e^{-x}$ $\qquad$ $f(x) = -e^{-x}$

$g(x) = x$ $\qquad$ $g'(x) = 1$

$$\int e^{-x}\,dx$$

$$= -xe^{-x} - \int -e^{-x}\,dx = -xe^{-x} + \int e^{-x}\,dx$$

$$= -xe^{-x} - e^{-x} + C$$

$$\int \frac{1}{x\ln(2x)}\,dx = \int \frac{1}{x} \cdot \frac{1}{\ln(2x)}\,dx = \int \frac{1}{y}\,dy$$

$\frac{1}{y}$ $\quad dy$

$$\ln(2x) = y$$

$$D[\ln(2x)]\,dx = D[y]\,dy$$

$$\frac{2}{2x}\,dx = dy$$

$$\boxed{\frac{1}{x}\,dx = dy}$$

$$= \ln|y| = \ln|\ln(2x)| + C$$