I parte   9 ÷ 15

1) $P(x) = 2x^4 - 3x^3 + x = x(2x^3 - 3x^2 + 1)$

$Q(x) = 2x^3 - 3x^2 + 1$

$m = \pm 1$

$n = \pm 1 \quad \pm 2$

$\frac{m}{n}$ $\quad \frac{\pm 1}{\pm 1} \quad \frac{\pm 1}{\pm 2}$

$\pm 1 \quad \pm \frac{1}{2}$

$Q(1) = 2 - 3 + 1 = 0$ quindi $Q(x)$ è divisibile per $(x-1)$

$Q(x) : (x-1)$

$$\begin{array}{rrrr|l}
2x^3 & -3x^2 & +0x & +1 & x-1 \\
\hline
-\cancel{2x^3} & +2x^2 & & & 2x^2 - x - 1 \\
\hline
 & -\cancel{x^2} & +0x & +1 & \\
 & +\cancel{x^2} & -x & & \\
\hline
 & & -x & +1 & \\
\end{array}$$

quoziente: $2x^2 - x - 1$

$$\begin{array}{r|}
+x-1 \\ \hline
0
\end{array}$$

resto $\rightarrow 0$

$$Q(x) : (x-1) = 2x^2 - x - 1$$

$$\Rightarrow \quad Q(x) = \underbrace{(2x^2 - x - 1)}_{Z(x)}(x-1)$$

$Z(x)$ è di 2° grado

$$2x^2 - x - 1 = 0 \qquad x_{1,2} = \frac{+1 \pm \sqrt{1+8}}{4} = \frac{+1 \pm 3}{4} \begin{cases} +1 \\ -\frac{1}{2} \end{cases}$$

$$2x^2 - x - 1 = 2(x-1)\left(x+\frac{1}{2}\right)$$

$$2x^4 - 3x^3 + x = x\,(\underbrace{2x^3 - 3x^2 + 1}_{Q(x)}) = x(x-1)(\underbrace{2x^2 - x - 1}_{Z(x)})$$

$$= x(x-1)\,2(x-1)\left(x+\frac{1}{2}\right) = 2x(x-1)^2\left(x+\frac{1}{2}\right)$$

gli zeri sono $\boxed{x = 0,\ x = 1,\ x = -\frac{1}{2}}$

$$2x(x-1)^2\left(x+\frac{1}{2}\right) = 0$$

$$x = 0 \quad \vee \quad (x-1)^2 = 0 \quad \vee \quad x + \frac{1}{2} = 0$$

2) $\dfrac{x}{y^4}\, e^{\frac{1}{y}} + \dfrac{2}{y^3}\, e^{\frac{2}{y}}$ $\qquad \dfrac{1}{y^4}\, e^{\frac{1}{y}}$

$$\left(\ ? \ \pm \ ? \ \right) \frac{1}{y^4}\, e^{\frac{1}{y}}$$

$$\left(x + 2ye^{\frac{1}{y}}\right)\frac{1}{y^4}e^{\frac{1}{y}}$$

$$2\frac{\cancel{y}}{y^4}e^{\frac{1}{y}}\cdot e^{\frac{1}{y}} = \frac{2}{y^3}e^{\frac{1}{y}+\frac{1}{y}} = \frac{2}{y^3}e^{\frac{2}{y}}$$

3)

$$2(x+1)\ln x + 3(x+1) = 0 \qquad D: x > 0$$

$$\frac{\cancel{2}\cancel{(x+1)}\ln x}{\cancel{2}\cancel{(x+1)}} = \frac{-3\cancel{(x+1)}}{2\cancel{(x+1)}}$$

$\cancel{x \neq}$

$x+1 \neq 0$

$x \neq -1$

$$\ln x = -\frac{3}{2}$$

$$\ln x = \ln e^{-\frac{3}{2}}$$

$$x = e^{-\frac{3}{2}} = \frac{1}{e^{\frac{3}{2}}}$$

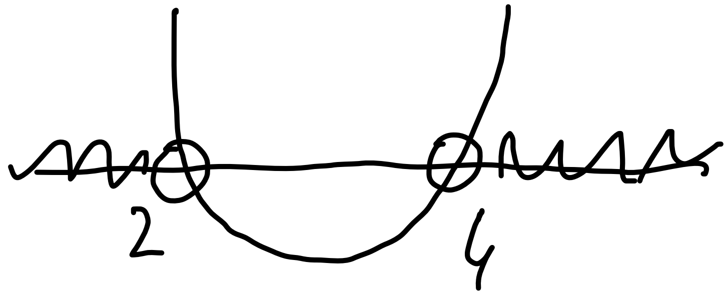
4)

$$x^4 - 6x^2 + 8 > 0 \qquad t = x^2$$

$$+\, t^2 - 6t + 8 > 0$$

$$t^2 - 6t + 8 = 0 \qquad t_{1,2} = \frac{6 \pm \sqrt{36-32}}{2} = \frac{6 \pm 2}{2} = \begin{cases} 4 \\ 2 \end{cases}$$

$$t_1 = 2 \,;\; t_2 = 4$$

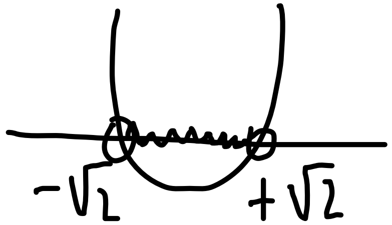


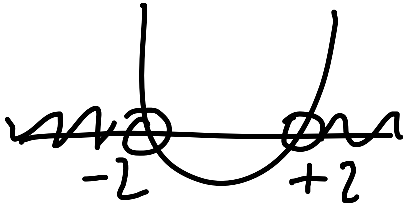
$$t < 2 \quad \vee \quad t > 4$$

$$x^2 < 2 \quad \vee \quad x^2 > 4$$

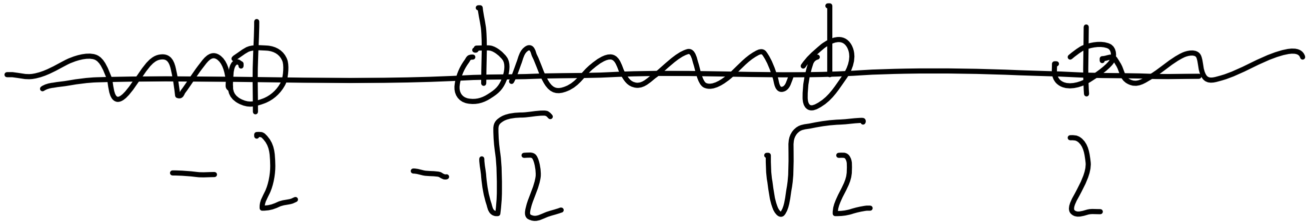
$$x^2 = 2 \qquad\qquad x^2 = 4$$

$x = \pm\sqrt{2}$ $\qquad$ $x = \pm 2$





$-\sqrt{2} < x < \sqrt{2} \quad \vee \quad x < -2 \ \vee \ x > 2$



$x < -2 \quad \vee \quad -\sqrt{2} < x < \sqrt{2} \quad \vee \quad x > 2$

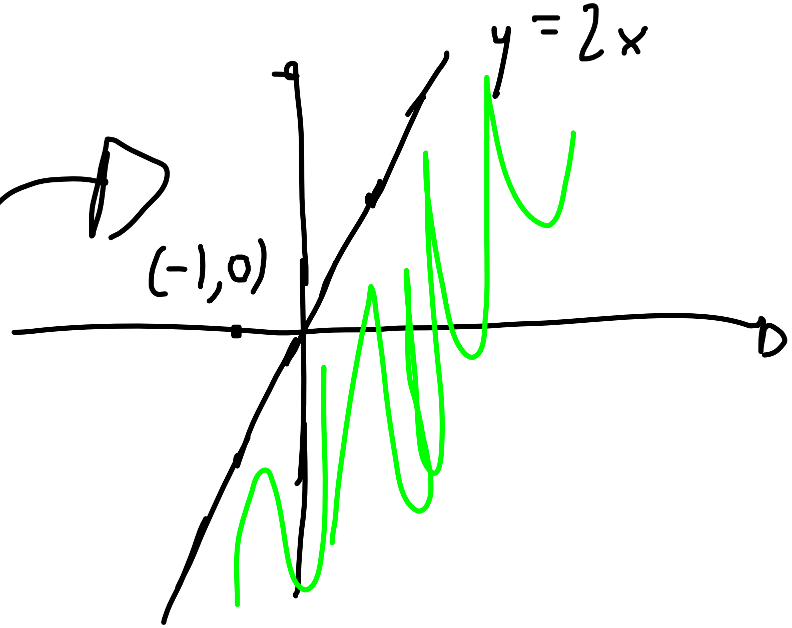
5) $4x^2 - y^2 > 0$ $\qquad$ $4x^2 - y^2 = 0$

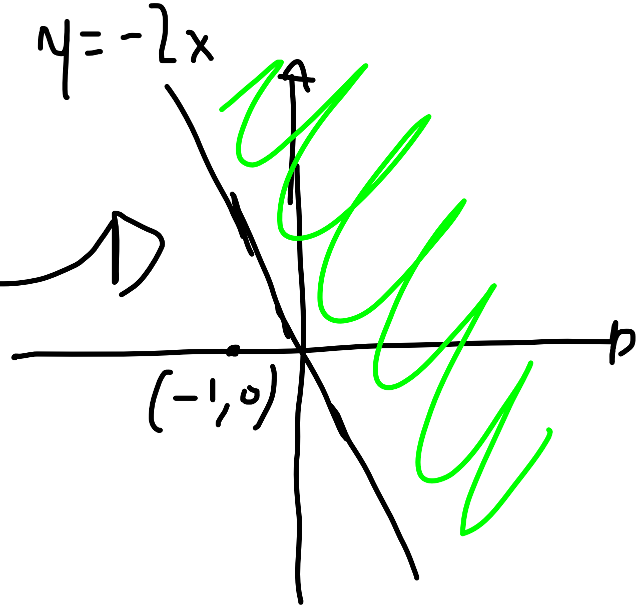
$(2x + y)(2x - y) > 0$

a) $\begin{cases} 2x+y>0 \\ 2x-y>0 \end{cases}$

v b) $\begin{cases} 2x+y<0 \\ 2x-y<0 \end{cases}$

a) $\begin{cases} \cancel{2x}\ y>-2x \\ y<+2x \end{cases}$





$y<2x \quad (-1,0)$

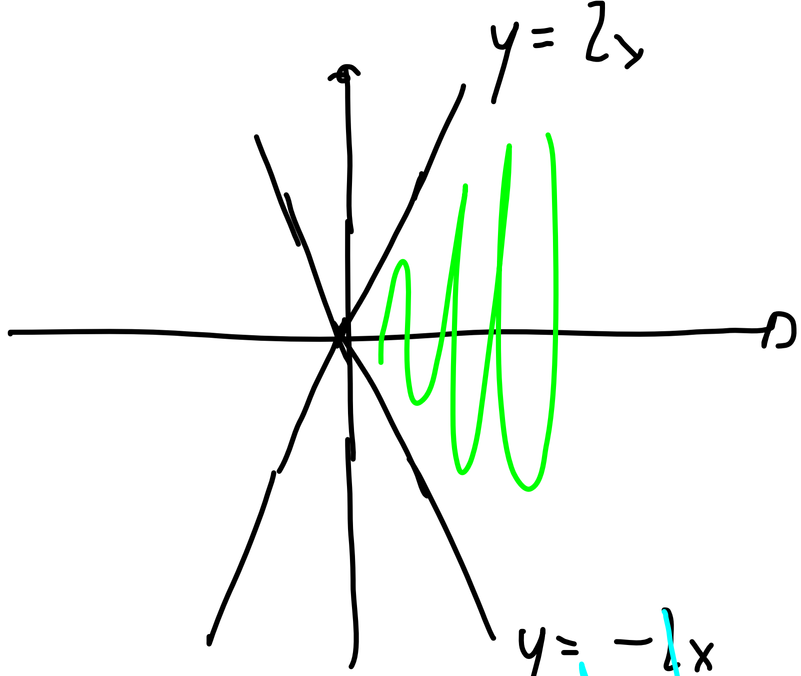
$0<2\cdot(-1)$

$0<-2$

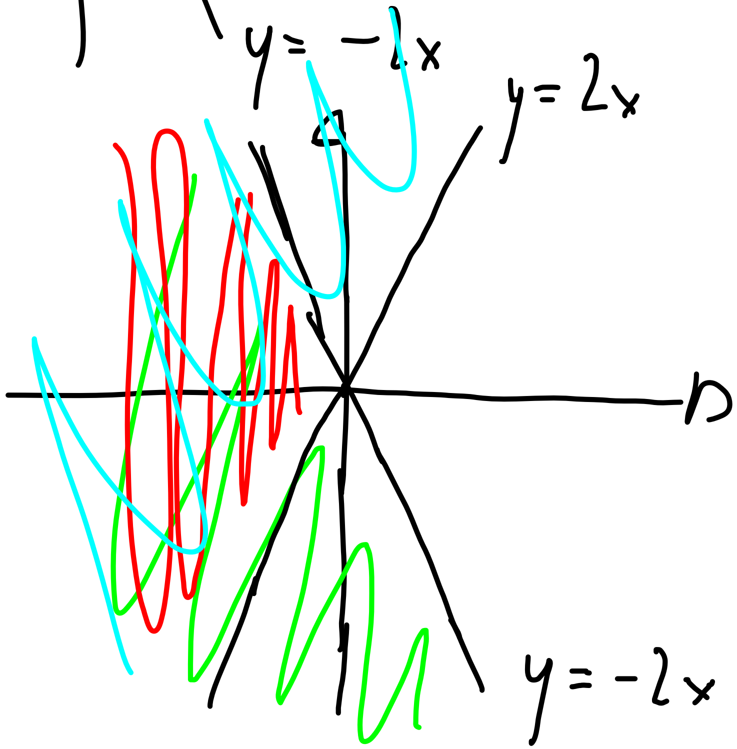
$\boxed{y>-2x}$

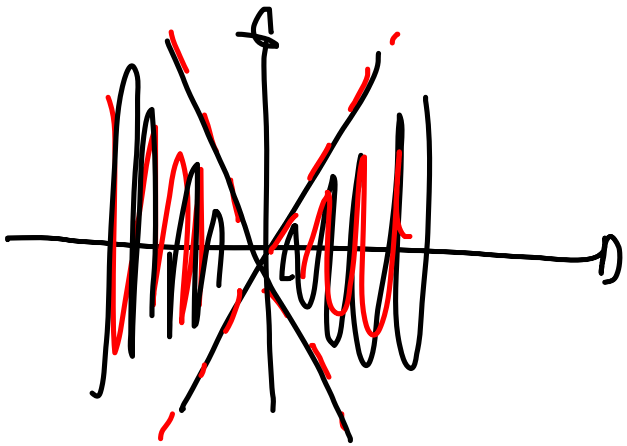
$0>-2(-1)$

$0 > 2$

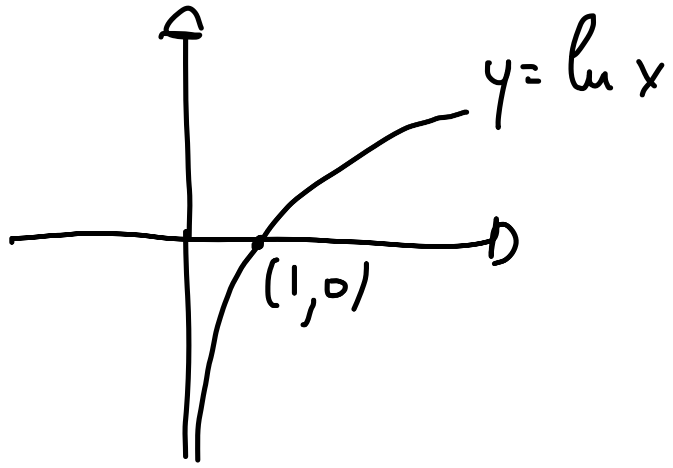


b) $\begin{cases} 2x + y < 0 \\ 2x - y < 0 \end{cases}$ $\begin{cases} y < -2x \\ y > 2x \end{cases}$

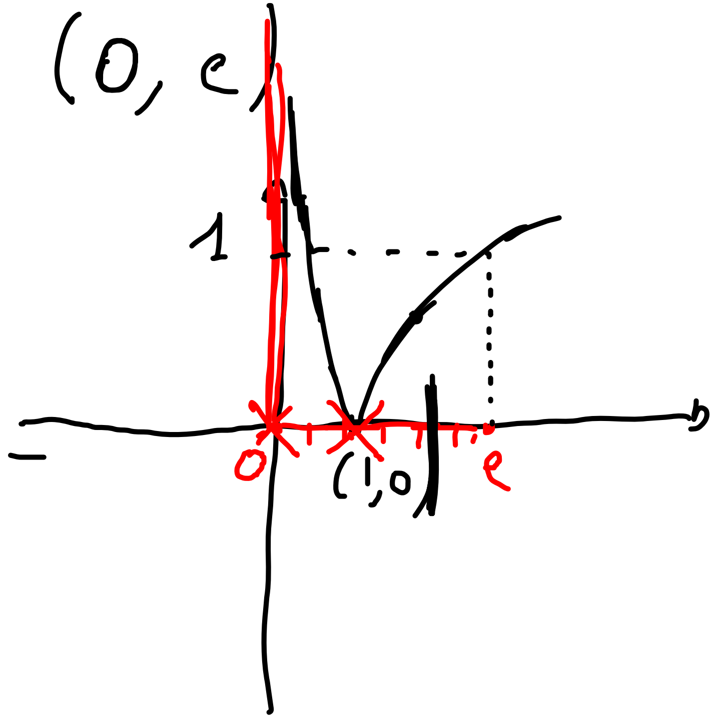




6) $f(x) = |\ln x|$



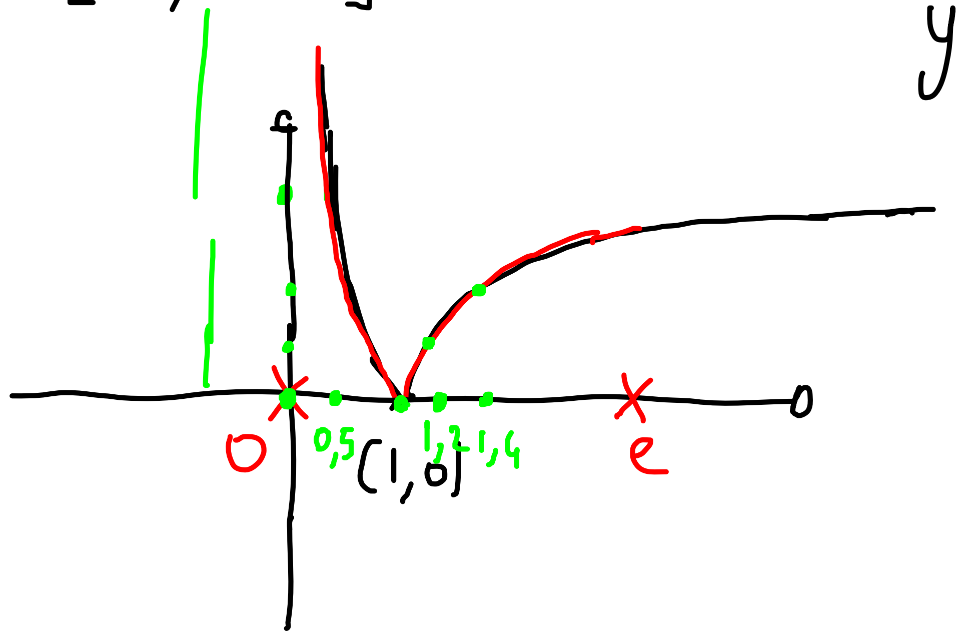
$(0, e)$



$x = e \quad y = ?$

$y = |\ln e| = 1$

$f(0,e)$ $\quad [0, +\infty]$



7) $$\lim_{x \to 0^+} \left(\underbrace{\frac{1}{\underbrace{\ln x}_{-\infty}}}_{0} + e^{\underbrace{-\overbrace{\tfrac{1}{x}}^{+\infty}}_{-\infty}} \right) = 0$$

8) $$f(x) = x\left(x + e^{-\frac{1}{x}}\right) = x^2 + xe^{-\frac{1}{x}}$$

$$D\left[x^2 + xe^{-\frac{1}{x}}\right] = 2x + D\left[xe^{-\frac{1}{x}}\right] =$$

$$= 2x + D[x]e^{-\frac{1}{x}} + x\,D\left[e^{-\frac{1}{x}}\right] =$$

$$= 2x + e^{-\frac{1}{x}} + x\left(+\frac{1}{x^2}\right)e^{-\frac{1}{x}}$$

$$D\left[e^{-\frac{1}{x}}\right] \qquad x \to -\frac{1}{x} = y \to e^{y}$$

$$D\left[-\frac{1}{x}\right] \cdot D\left[e^{y}\right] = \frac{1}{x^2}\, e^{-\frac{1}{x}}$$

9) $$\sum_{n=0}^{\infty} 3^{-n/2} = \sum_{n=0}^{\infty} \left(3^{-\frac{1}{2}}\right)^{n} = \sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{3}}\right)^{n} \qquad r = \frac{1}{\sqrt{3}} < 1$$

$$\frac{1}{1-r} = \frac{1}{1-\frac{1}{\sqrt{3}}} = \frac{1}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{3}-1}$$

10) $$f(x,y) = y \ln\left(\frac{y}{1-x}\right) \qquad \nabla f = \left(\frac{df}{dx}, \frac{df}{dy}\right)$$

$$\frac{df}{dx} = \cancel{\frac{dy}{dx} \cdot \ln(\ )} + y \cdot \frac{d}{dx}\left[\ln\left(\frac{y}{1-x}\right)\right]$$

$$= y \cdot \frac{d}{dx}\left(\frac{y}{1-x}\right) \cdot \frac{1}{\frac{y}{1-x}} = y \cdot \frac{d}{dx}\left(y \cdot \frac{1}{1-x}\right) \cdot \frac{1-x}{y} =$$

$$= y \cdot \cancel{y} \cdot \underbrace{\frac{d}{dx}\left(\frac{1}{1-x}\right)} \cdot \frac{1-x}{\cancel{y}} = y \cdot \underbrace{\left(-\frac{(-1)}{(1-x)^2}\right)} \cdot (1-x)$$

$$= \frac{y}{1-x}$$

$$\frac{d}{dz} \ln z = \frac{1}{z}$$

$$\frac{df}{dy} = 1 \cdot \ln\left(\frac{y}{1-x}\right) + y \cdot \frac{d}{dy}\left[\ln\left(\frac{y}{1-x}\right)\right] =$$

$$= \ln\left(\frac{y}{1-x}\right) + y \cdot \frac{d}{dy}\left(\frac{y}{1-x}\right) \cdot \frac{1}{\frac{y}{1-x}}$$

$$\cancel{\frac{df}{dx} =}$$

$$D\left[\frac{f(x)}{g(x)}\right] = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$= \ln\left(\quad\right) + y \cdot \left(\frac{1 \cdot (1-x) - y \cdot 0}{(1-x)^2}\right) \cdot \frac{1-x}{y} =$$

$$= \ln\left(\quad \right) + \underbrace{\cancel{x}\left(\frac{1-\cancel{x}}{(1-\cancel{x})^2}\right)\frac{(\cancel{1}-x)}{\cancel{x}}}_{1} = \ln\left(\quad \right) + 1$$

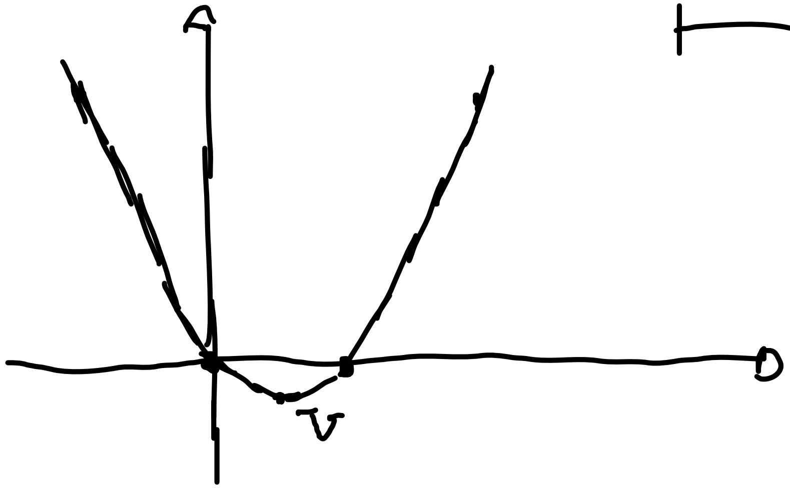
II parte

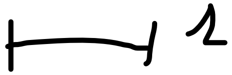
ES. 1

$$f(x) = \begin{cases} x^2 - x & P \quad x \leq 0 \\ e^{-x} - 1 & x > 0 \end{cases}$$

$P: y = x^2 - x$ $\qquad x_v = -\frac{b}{2a} = \frac{1}{2} \qquad v\left(\frac{1}{2}, -\frac{1}{4}\right)$

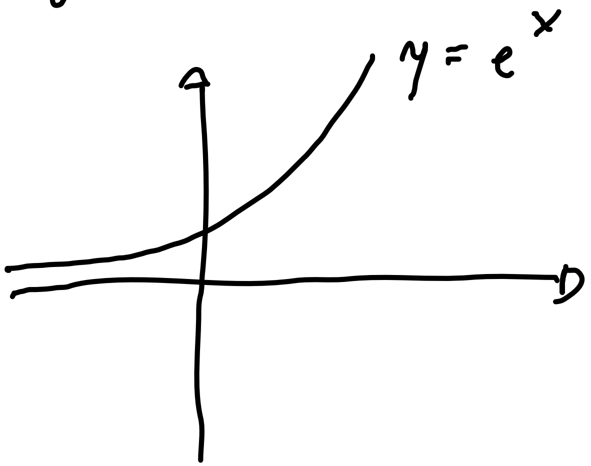
$$y_v = \left(\frac{1}{2}\right)^2 - \frac{1}{2} = -\frac{1}{4}$$

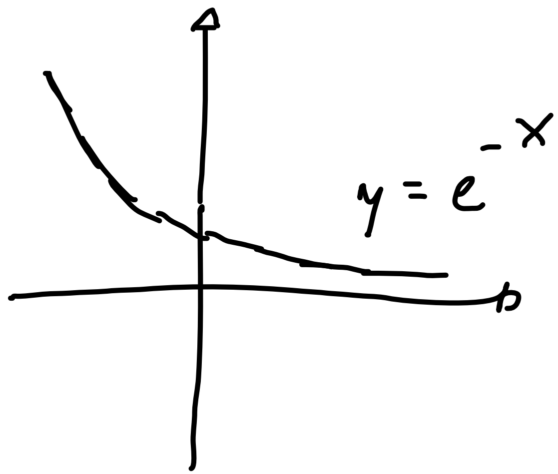


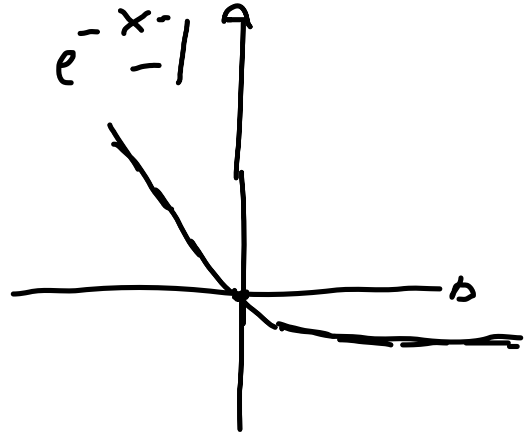


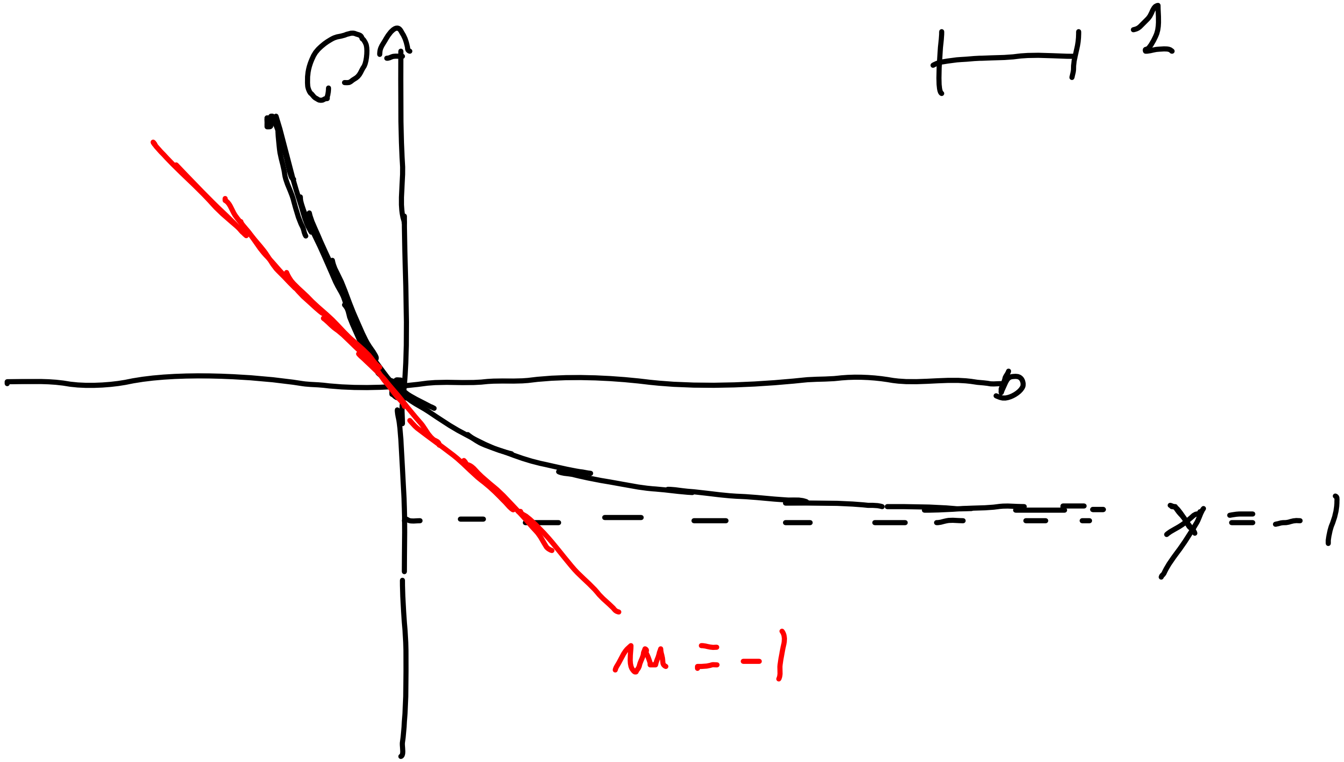
x	y
0	0
1	0
-1	2
2	2

$y = e^{-x} - 1$









$\mathrm{Im}\, f = (-1, +\infty)$

NO MAX / MIN

f continua

$$f'(x) = \begin{cases} 2x-1 & x<0 \\ -e^{-x} & x>0 \end{cases}$$

$$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^+} f'(x)$$

$$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} (2x-1) = -1$$

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} (-e^{-x}) = -1$$

la f è derivabile in $x = 0$

ES 2

$$A = \begin{array}{c} \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} \end{array}$$

ha rango 3?

$$\det A = 2 \cdot \det \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} - 1 \cdot \det \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} =$$

$$= 2(2-1) - 1(2-0) = 2 \cdot 1 - 1 \cdot 2 = 2 - 2 = 0$$

$$\det A = 0 \Rightarrow r(A) < 3$$

A ha rango 2

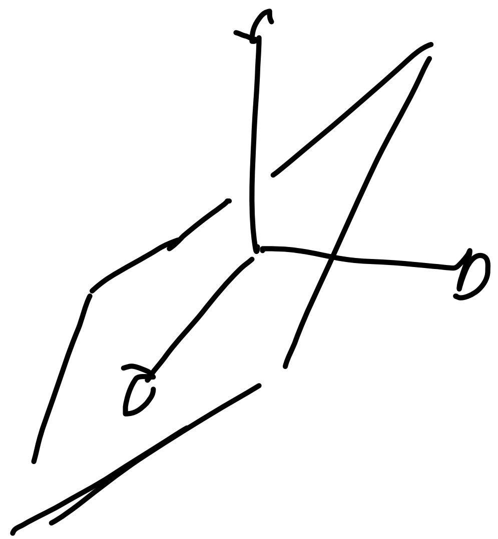
$a v_1 + b v_2 = v_3$ per qualche $a, b \neq (0,0)$

$$a \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{cases} 2a + b = 0 \\ a + b = 1 \\ b = 2 \end{cases} \qquad \begin{cases} 2(-1) + 2 = 0 \quad \text{OK!} \\ a = 1 - b = 1 - 2 = -1 \\ b = 2 \end{cases}$$

$a = -1$

$b = 2$



le colonne di A generano sottospazio di $\mathbb{R}^3$ di

$\dim = 2 \quad B = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

$$e^1 = (1, 0, 0)$$

$$a v_1 + b v_2 = e^1 \qquad e^1 \notin S'$$

$$\begin{cases} 2a + b = 1 \\ a + b = 0 \\ b = 0 \end{cases} \qquad \begin{cases} 0 = 1 \\ a = 0 \\ b = 0 \end{cases} \qquad \text{impossibile}$$

$$\det A = 0 \begin{cases} \text{semi.def. positiva} \\ \text{semidef. negativa} \\ \text{indefinita} \end{cases}$$

Ordine 1

$$\begin{pmatrix} \boxed{2} & 1 & 0 \\ 1 & \boxed{1} & 1 \\ 0 & 1 & \boxed{2} \end{pmatrix} \geq 0$$

Ordine 2

$1^\circ\, 2^\circ\, r$ $\qquad$ $1^\circ\, 3^\circ\, r$ $\qquad$ $2^\circ\, 3^\circ\, r$ ≥ 0

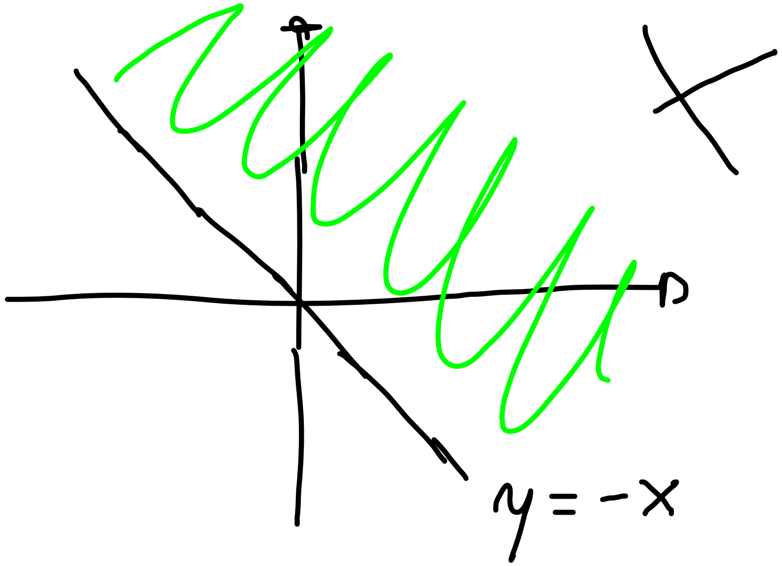
$$\det\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = 1 \qquad \det\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 4 \qquad \det\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = 1$$

$\boxed{\text{Semidefinita positiva}}$

3) $$f(x,y) = \ln(x+y) \cdot \ln(1-x^2-y)$$

$$\begin{cases} x+y > 0 & \text{a)} \\ 1-x^2-y > 0 & \text{b)} \end{cases}$$

e) $y > -x$

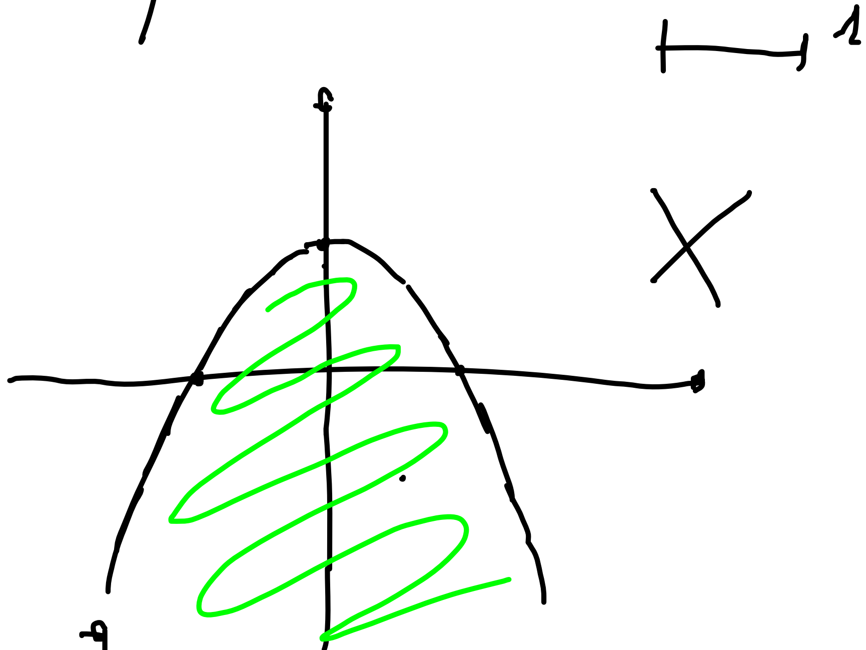


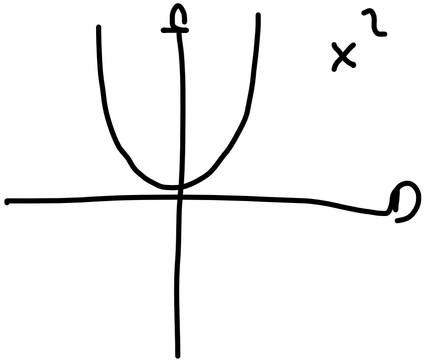
b) $1 - x^2 - y > 0$

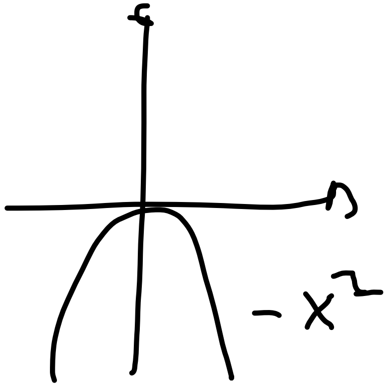
$-y > -1 + x^2$

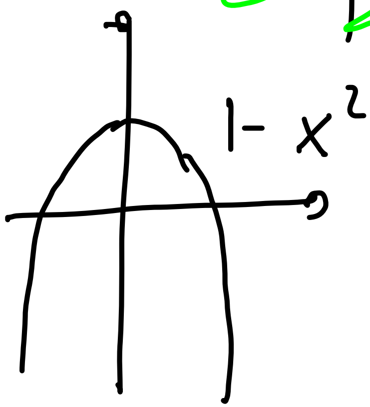
$\boxed{y < 1 - x^2}$ $\quad 0 < 1$

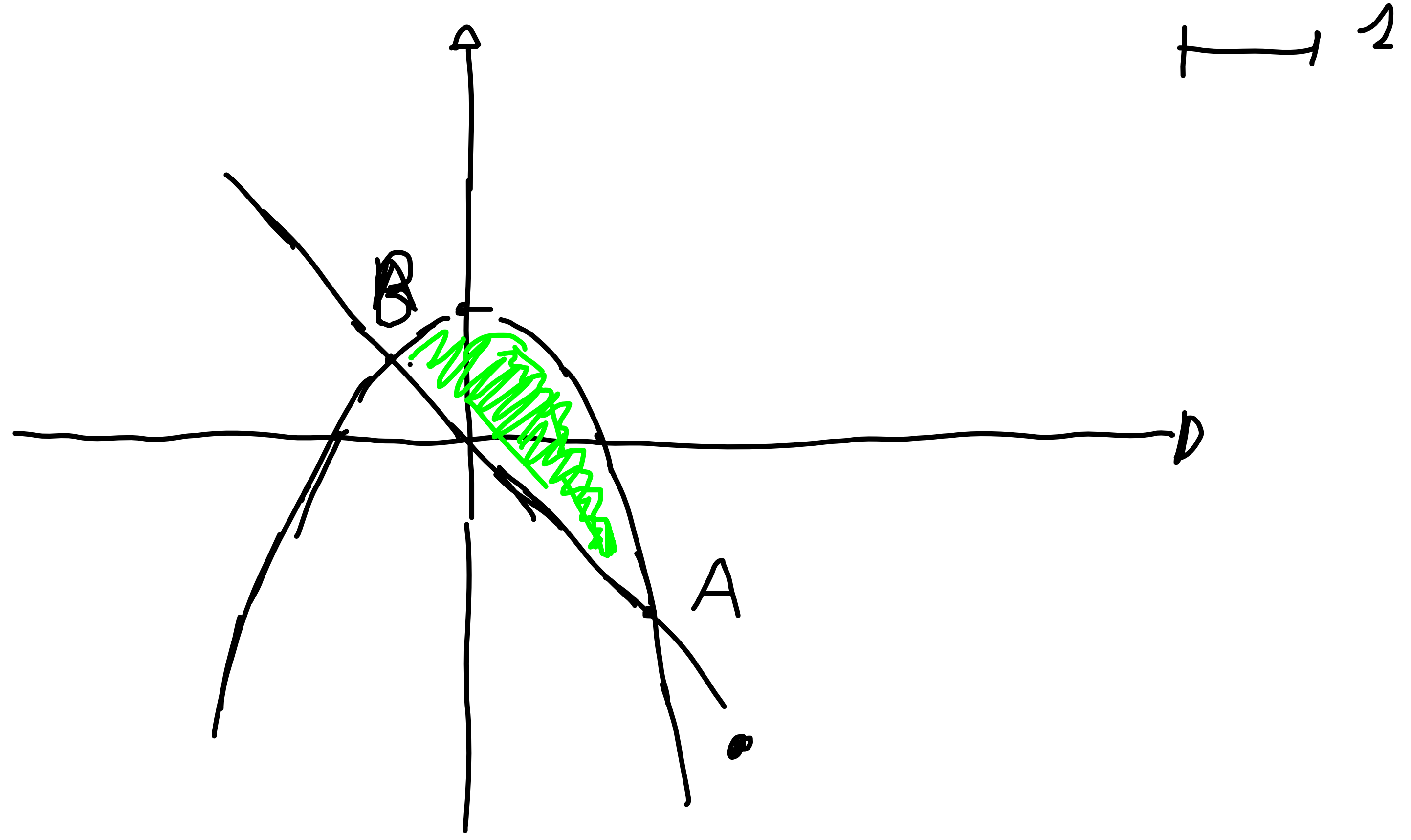
$y = 1 - x^2$











$$\begin{cases} y = -x \\ y = 1 - x^2 \end{cases}$$

$$A\left(\frac{1+\sqrt{5}}{2}, -\frac{1+\sqrt{5}}{2}\right)$$

$$B\left(\frac{1-\sqrt{5}}{2}, -\frac{1-\sqrt{5}}{2}\right)$$

$$\frac{d}{dy} = \frac{d}{dy}\left[\ln(x+y)\right] \cdot \ln(1-x^2-y) + \ln(x+y) \cdot \frac{d}{dy}\left[\ln(1-x^2-y)\right]$$

$$= 1 \cdot \frac{1}{x+y} \cdot \ln(1-x^2-y) + \ln(x+y) \cdot (-1) \cdot \frac{1}{1-x^2-y}$$

$$= \frac{1}{x+y} \ln(1-x^2-y) - \frac{1}{1-x^2-y} \cdot \ln(x+y)$$

$\left(\frac{1}{2}, \frac{1}{2}\right)$ è stazionario?

$$\frac{df}{dy} \overset{?}{=} 0$$

$$\frac{df}{dy}\left(\frac{1}{2}, \frac{1}{2}\right) = 0$$

$$\frac{df}{dx}\left(\frac{1}{2}, \frac{1}{2}\right) = 0$$