I parte - P. CONCLUSIVA 8:30

1) $$\int_1^e \frac{\sqrt[4]{1+\ln x}}{x}\,dx$$

$$\int \frac{1}{x}\cdot(1+\ln x)^{4}\,dx = \int f'(x)\cdot f(x)^{4}\,dx = \frac{f(x)^{\frac{1}{4}+1}}{\frac{1}{4}+1} =$$

$$f(x) = 1+\ln x$$

$$f'(x) = \frac{1}{x}$$

$$= \frac{(1+\ln x)^{\frac{5}{4}}}{\frac{5}{4}} \neq \frac{4}{5}\sqrt[4]{(1+\ln x)^5} + c$$

$$\int_1^e \frac{\sqrt[4]{(1+\ln x)}}{x}\,dx = \left[\frac{4}{5}\sqrt[4]{(1+\ln x)^5}\right]_1^e = \frac{4}{5}\sqrt[4]{(1+\ln e)^5} - \frac{4}{5}\sqrt[4]{(1+\ln 1)^5} =$$

$$= \frac{4}{5}\sqrt[4]{(1+1)^5} - \frac{4}{5}$$

2) $$\int \sqrt[4]{x}\,\ln x\,dx = \int x^{\frac{1}{4}} \ln x\,dx = (*)$$

$$\cancel{D}\int [f(x)g(x)] = \int f'(x)g(x) + \int f(x)g'(x)$$

$$\int f'(x)g(x) = f(x)g(x) - \int f(x)g'(x)$$

$$f'(x) = x^{\frac{1}{4}} \qquad f(x) = \frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} = \frac{x^{\frac{5}{4}}}{\frac{5}{4}} = \frac{4}{5}x^{\frac{5}{4}}$$

$$g(x) = \ln x \qquad g'(x) = \frac{1}{x}$$

$$(*) = \frac{4}{5}x^{\frac{5}{4}}\ln x - \int \frac{4}{5}x^{\frac{5}{4}} \cdot \frac{1}{x}\,dx =$$

$$= \frac{4}{5}x^{\frac{5}{4}}\ln x - \frac{4}{5}\int x^{\frac{5}{4}-1}\,dx = \frac{4}{5}x^{\frac{5}{4}}\ln x - \frac{4}{5}\int x^{\frac{1}{4}}\,dx =$$

$$= \frac{4}{5}x^{\frac{5}{4}}\ln x - \frac{4}{5}\,\frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} = \frac{4}{5}x^{\frac{5}{4}}\ln x - \frac{4}{5}\cdot\frac{4}{5}x^{\frac{5}{4}} =$$

$$= \frac{4}{5}x^{\frac{5}{4}}\ln x - \frac{16}{25}x^{\frac{5}{4}} = \frac{4}{5}x^{\frac{5}{4}}\left(\ln x - \frac{4}{5}\right) + C$$

3) $$\int_1^{+\infty} \frac{1}{x^5}\,dx = \lim_{a\to+\infty}\int_1^a \frac{1}{x^5}\,dx = \lim_{a\to+\infty}\left[-\frac{1}{4x^4}\right]_1^a =$$

$$\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx = \frac{x^{-5+1}}{-5+1} = -\frac{1}{4}x^{-4} = -\frac{1}{4x^4}$$

$$= \lim_{e \to +\infty} \left[-\frac{1}{4e^4} - \left(-\frac{1}{4} \right) \right] = \lim_{e \to +\infty} \left[\underbrace{-\frac{1}{4e^4}}_{0} + \frac{1}{4} \right] = \frac{1}{4}$$

4) $$\sum_{m=0}^{\infty} 2^{-m/3} = \sum_{n=0}^{\infty} \left(2^{-\frac{1}{3}} \right)^m = \sum_{n=0}^{\infty} \left(\frac{1}{2^{\frac{1}{3}}} \right)^n = \sum_{n=0}^{\infty} \left(\frac{1}{\sqrt[3]{2}} \right)^n$$

$r = \frac{1}{\sqrt[3]{2}} \qquad |r| < 1 \Rightarrow$ converge

$$\frac{1}{1-r} = \frac{1}{1 - \frac{1}{\sqrt[3]{2}}} = \frac{1}{\frac{\sqrt[3]{2} - 1}{\sqrt[3]{2}}} = \frac{\sqrt[3]{2}}{\sqrt[3]{2} - 1}$$

5) $$\sum_{n=1}^{+\infty} \frac{\sqrt{n} + \ln n}{\sqrt{n^3} + n}$$

$$\lim_{n \to +\infty} a_n \begin{cases} = 0 & \text{può convergere} \\ \neq 0 & \text{non converge} \end{cases}$$

$$\lim_{n \to +\infty} a_n = \lim_{n \to +\infty} \frac{\sqrt{n} + \cancel{\ln n}}{\sqrt{n^3} + \cancel{n}} = \lim_{n \to +\infty} \frac{n^{\frac{1}{2}}}{n^{\frac{3}{2}}} =$$

$$n^{\frac{3}{2}} \quad n^{1}$$

$$\lim_{n \to +\infty} \frac{1}{n^{\frac{3}{2}-\frac{1}{2}}} = \lim_{n \to +\infty} \frac{1}{n} = 0 \quad \text{può convergere!}$$

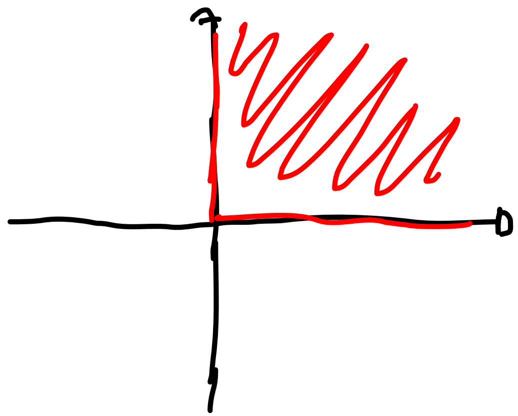
$$\frac{\sqrt{n} + \ln n}{\sqrt{n^3} + n} \sim \frac{1}{n} \Rightarrow \text{diverge}$$

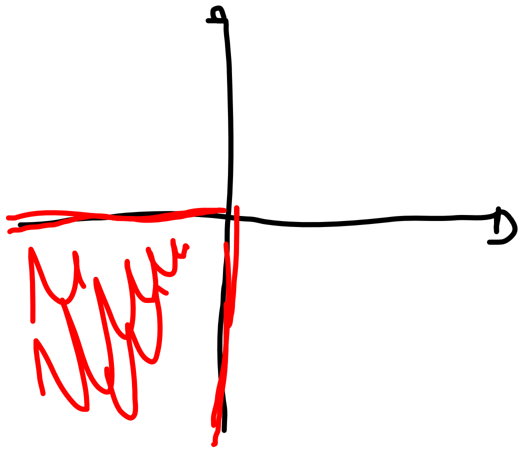
diverge

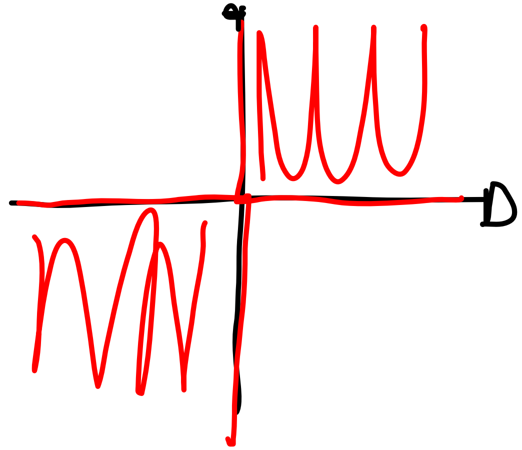
7) $A = \left(\begin{array}{c|cc|c} 1 & -2 & 2 & -1 \\ -2 & 4 & 1 & 2 \end{array}\right)$ $\quad r(A) = 2$

8) $f(x,y) = \sqrt{xy}$ $\quad xy \geq 0$

$\begin{cases} x \geq 0 \\ y \geq 0 \end{cases} \vee \begin{cases} x \leq 0 \\ y \leq 0 \end{cases}$







9) $f(x,y) = xe^{\frac{1}{y}}$ $\qquad \nabla f = \left(\frac{df}{dx}, \frac{df}{dy}\right)$

$$\frac{df}{dx} = e^{\frac{1}{y}}$$

$$\frac{df}{dy} = \cancel{\frac{d}{dy}[x]\cdot e^{\frac{1}{y}}} + x\cdot\frac{d}{dy}\left[e^{\frac{1}{y}}\right]$$

$$= x\cdot\frac{d}{dy}\left(\frac{1}{y}\right)\cdot e^{\frac{1}{y}} = x\left(-\frac{1}{y^2}\right)e^{\frac{1}{y}} = -\frac{x}{y^2}e^{\frac{1}{y}}$$

10) $Q(x,y) = 2xy - 2y^2$

$$A = \begin{array}{cc} x^2 & \frac{xy}{2} \\ \multicolumn{2}{c}{\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}} \\ \frac{xy}{2} & y^2 \end{array}$$

$\det = 0 - 1 = -1 < 0$ INDEFINITA

$Q(x,y) = 2xy - 2y^2$

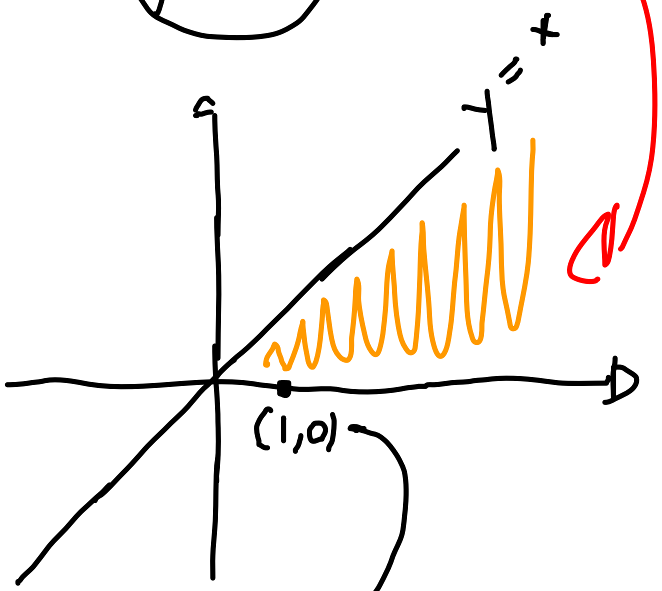
$2xy - 2y^2 > 0$

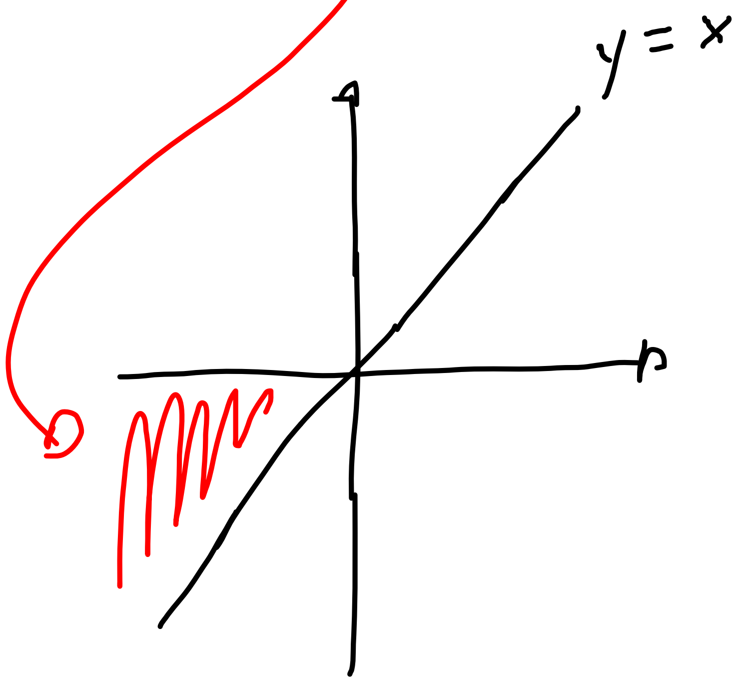
$2y(x-y) > 0$

$$\begin{cases} 2y > 0 \\ x - y > 0 \end{cases} \vee \begin{cases} 2y < 0 \\ x - y < 0 \end{cases}$$

$$\begin{cases} y > 0 \\ -y > -x \end{cases} \vee \begin{cases} y < 0 \\ -y < -x \end{cases}$$

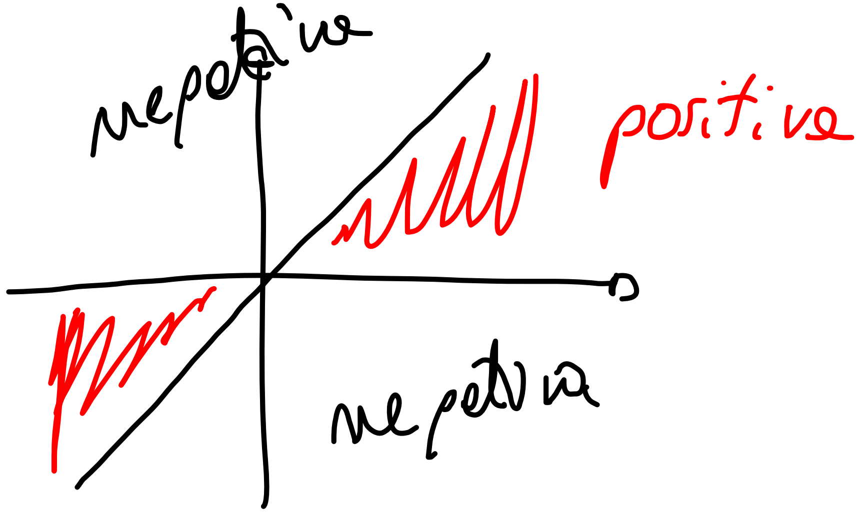
$$\begin{cases} y > 0 \\ y < x \end{cases} \cup \begin{cases} y < 0 \\ y > x \end{cases}$$





$y < x$

$0 < 1$ vera



INDEFINITA

6)

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\begin{matrix} 0 & -1 \\ 1 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 \\ 1 & -1 \end{matrix}$$

$$A_{adj}A = \begin{pmatrix} +1\cdot(-1) & -1\cdot(-1) & +1\cdot 1 \\ -1\cdot 1 & +1\cdot 1 & -1\cdot(-1) \\ +1\cdot 1 & -1\cdot 1 & 1\cdot(-1) \end{pmatrix}$$

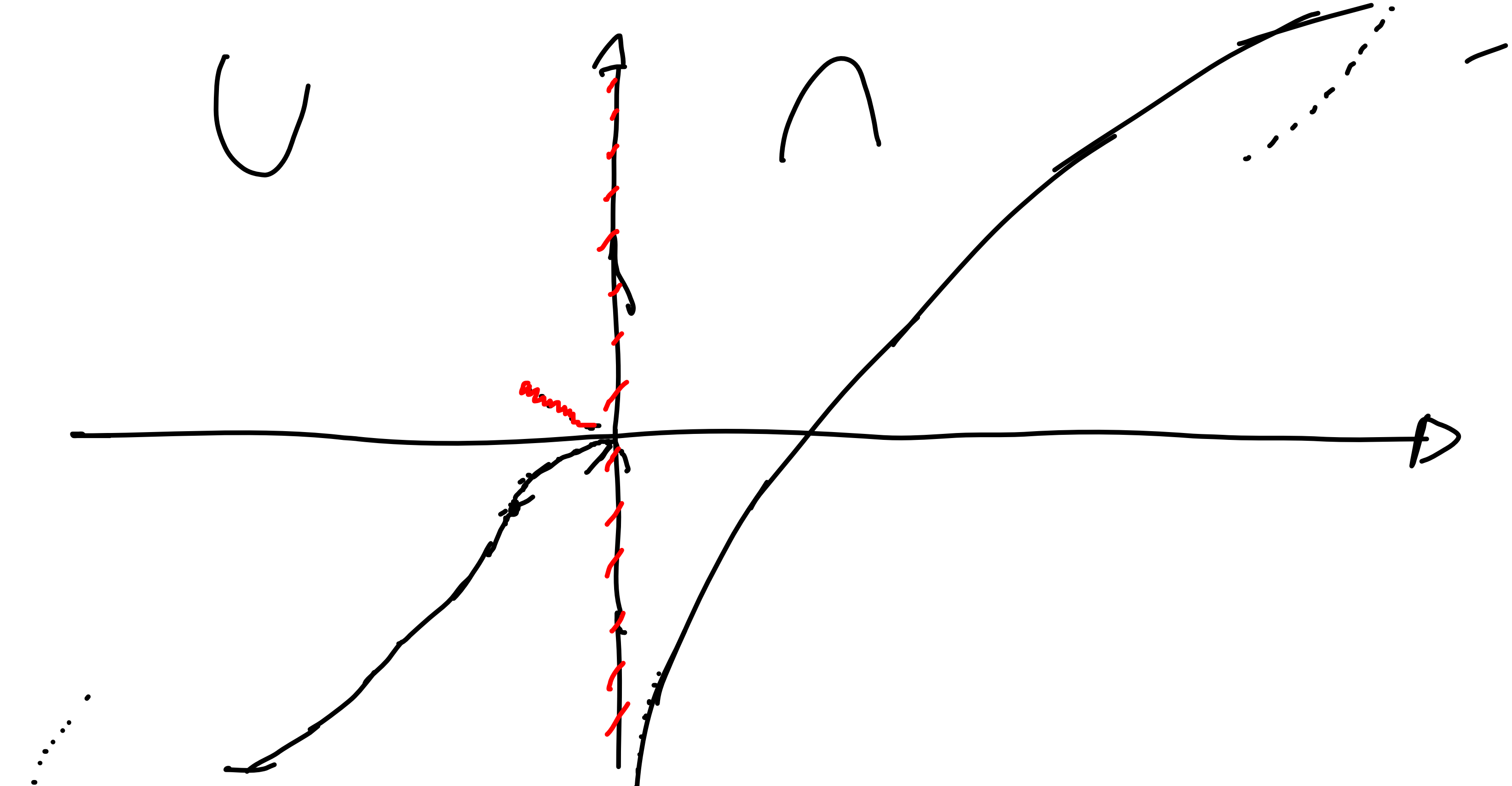
$$= \begin{matrix} -1 & +1 & +1 \\ -1 & +1 & +1 \\ +1 & -1 & -1 \end{matrix}$$

03/09/14 II parte

Es. 1

$$f(x) = x - e^{\frac{1}{x}}$$

$D: x \neq 0$



$-\infty \quad 0^+ \quad 0^+ \quad +\infty$

$$\lim_{x \to -\infty} \left(\underbrace{x}_{-\infty} - \underbrace{e^{\frac{1}{x} \to 0}}_{1} \right) = -\infty$$

$$\lim_{x \to 0^-} \left(\underbrace{x}_{0^-} - \underbrace{e^{\frac{1}{x} \to -\infty}}_{\to 0^+} \right) = 0$$

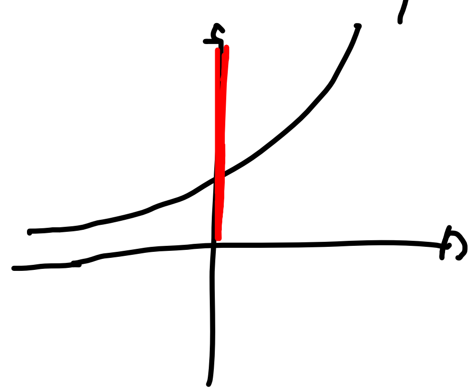
$$\lim_{x \to 0^+} \underbrace{\left(x - \underbrace{e^{\frac{1}{x} \to +\infty}}_{+\infty} \right)}_{-\infty} = -\infty$$

$$\lim_{x \to +\infty} \left(\underbrace{x}_{+\infty} - \underbrace{e^{\frac{1}{x} \to 0}}_{1} \right) = +\infty$$

$$f'(x) = 1 - D\left[e^{\frac{1}{x}}\right] = 1 - \left(-\frac{1}{x^2}\right) e^{\frac{1}{x}} = 1 + \frac{1}{x^2} e^{\frac{1}{x}}$$

$$f'(x) \geq 0 \qquad 1 + \underset{+}{\frac{1}{x^2}} \underset{+}{e^{\frac{1}{x}}} \geq 0$$

$$\forall x$$



f é sempre crescente

$$f''(x) = -\frac{2x}{x^4} e^{\frac{1}{x}} + \frac{1}{x^2}\left(-\frac{1}{x^2}\right) e^{\frac{1}{x}} = -\frac{2x}{x^4} e^{\frac{1}{x}} - \frac{1}{x^4} e^{\frac{1}{x}} =$$

$$= \frac{(-2x-1)}{\underset{+}{x^4}} \underset{+}{e^{\frac{1}{x}}}$$

$f'' \geq 0 \qquad -2x-1 \geq 0$

$-2x \geq 1$

$x \leq -\frac{1}{2}$

	$-\frac{1}{2}$	
f	$\cup$	$\cap$
f''	$+$	$-$

$f\left(-\frac{1}{2}\right) = -\frac{1}{2} - e^{\frac{1}{-\frac{1}{2}}} =$

$= -\frac{1}{2} - \frac{1}{e^2} < 0$

$\left(-\frac{1}{2} ; -\frac{1}{2} - \frac{1}{e^2}\right)$

pendenza retta tangente in $x_0 = 1$

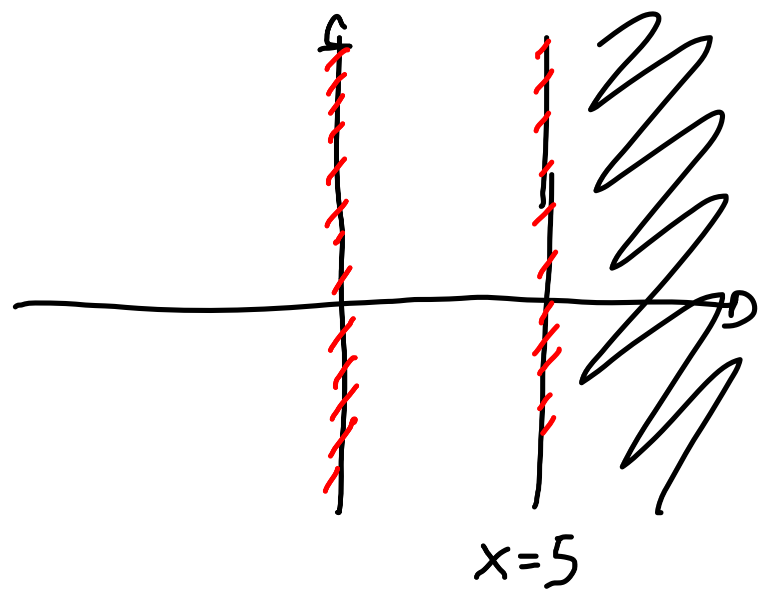
$m = f'(x_0) = f'(1) = 1 + \frac{1}{1} e^1 = 1 + e$

$y - y_0 = m(x - x_0)$

x_0

$y_0 = f(x_0)$

$m = f'(x_0)$



$-\infty$

0^-

0^+

5^-

ane y $\quad f(0)$

ane x $\quad f(x) = 0$

$$\int_1^2 \frac{f(x) - x}{x^2}\,dx = \int_1^2 \frac{x - e^{\frac{1}{x}} - x}{x^2}\,dx = \int_1^2 -\frac{1}{x^2} e^{\frac{1}{x}}\,dx$$

$$= \int_1^2 f'(x)\, e^{f(x)}\,dx = \left[e^{f(x)} \right]_1^2 = \left[e^{\frac{1}{x}} \right]_1^2 = e^{\frac{1}{2}} - e$$

ES. 2

$$A = \begin{array}{c} \\ w_1 \\ w_2 \\ w_3 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

$$\det \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} = 1 \det \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = 1(-1) = -1 \neq 0$$

$$\Rightarrow r(A) = 3$$

- 3 righe l. i. Le righe generano uno sottospazio di $\mathbb{R}^4$ di dimensione 3. $\mathcal{B} = \left\{ \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

- Le colonne sono linear. dipendenti. Generano sottospazio di $\mathbb{R}^3$ di dimensione 3. $\mathcal{B} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$$\overset{\times}{a v_1} + \overset{\times}{b v_2} + \overset{\times}{c v_4} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} a \cdot 0 + b + c \cdot 0 = 1 \\ a + b \cdot 0 + c \cdot 0 = 1 \\ a + b + c = 1 \end{cases} \quad \begin{cases} b = 1 \\ a = 1 \\ 1 + 1 + c = 1 \end{cases} \quad \begin{cases} b = 1 \\ d = 1 \\ c = -1 \end{cases}$$

$\Rightarrow (1,1,1) \in \mathcal{S}$ generato dalle colonne

? $= (0,0,0,1) \in \mathcal{S}$ generato dalle righe di A

$$a w_1^T + b w_2^T + c w_3^T = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$a \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 1 \\ \cancel{0} \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{cases} b + c = 0 \\ a + c = 0 \\ -a + b = 0 \\ c = 1 \end{cases} \qquad \begin{cases} b = -1 \\ a = -1 \\ -(-1) - 1 = 0 \text{ ok!} \\ c = 1 \end{cases}$$

$a = -1$

$b = -1$

$c = 1$

$\Rightarrow (0,0,0,1) \in S'$ generato delle righe di A

ES. 3

$$f(x,y) = \frac{1}{xy} - \ln\left(4 - x^2 - (y-2)^2\right)$$

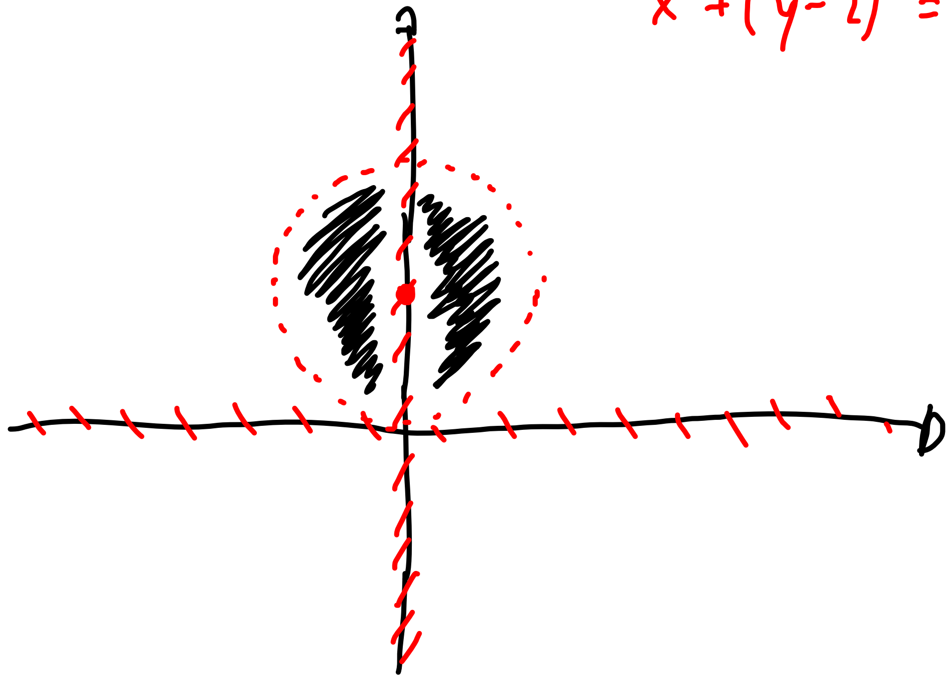
$x^2 + (y-2)^2 = 4$

D: $\begin{cases} x \neq 0 & x \\ y \neq 0 & x \\ 4 - x^2 - (y-2)^2 > 0 & x \end{cases}$ c)

c) $4 - x^2 - (y-2)^2 > 0$

$-x^2 - (y-2)^2 > -4$

$\boxed{x^2 + (y-2)^2 < 4}$



$C(0,2) \quad r = 2$

$$\nabla f = \left(\frac{df}{dx} , \frac{df}{dy} \right)$$

$$\frac{df}{dx} = \frac{d}{dx}\left(\frac{1}{xy}\right) - \frac{d}{dx}\left[\ln\left(4 - x^2 - (y-2)^2\right) \right]$$

$$= -\frac{y}{x^2y^2} - \left(-2x \cdot \frac{1}{4 - x^2 - (y-2)^2} \right) = -\frac{1}{x^2y} + \frac{2x}{4 - x^2 - (y-2)^2}$$

$$\frac{df}{dy} = \frac{d}{dy}\left(\frac{1}{xy}\right) - \frac{d}{dy}\left[\ln\left(4 - x^2 - (y-2)^2\right) \right]$$

$$(y-2)^2 = y^2 - 4y + 4$$

$$2(y-2)$$

$$= -\frac{x}{x^2y^2} - \left(-2(y-2) \cdot \frac{1}{4 - x^2 - (y-2)^2} \right)$$

$$= -\frac{1}{xy^2} + \frac{2(y-2)}{4 - x^2 - (y-2)^2}$$